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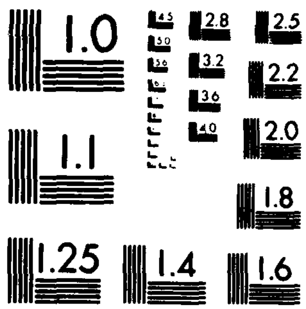

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RAY TRACING TECHNIQUES - DERIVATION AND APPLICATION TO ATMOSPHERIC SOUND PROPAGATION

by

S. David Roth

The Noise Control Laboratory

The Pennsylvania State University

January 1980

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ABSTRACT

It is commonly known that in non-homogeneous media (phase velocity dependent on location) refraction of acoustic signals occurs. Solving the wave equation with variable c is extremely involved and the cases where solutions can be found do not give very much insight into the physical meaning of the problem.

The method of ray tracing, the solution of the eikonal equation, readily adapts itself to non-homogeneous media and describes the propagation of wavefronts. It has been used extensively in underwater acoustics but not so much in atmospheric applications. Some reasons for the limited use of ray tracing techniques in outdoor sound propagation are that 1) most acoustic work in recent years has been for underwater applications due to Navy sponsoring, 2) and also that very few simultaneous measurements of acoustical and meteorological data have been performed.

Atmospheric sound ranging techniques have in the past neglected verticle velocity gradients. Ray tracing is a useful method in studying propagation in air and can be used as an adjustment to sound ranging methods to consider atmospheric variations.

Presented here is a derivation of the eikonal equation and its solution with an attempt to give physical reasons for this approach. A computer model of the technique of ray tracing for atmospheric applications (also an eigenray model) has been developed and some results are given using data collected in field measurements.

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List of Symbols

A - Amplitude

A - a constant = $\cos \theta_o$

Alpha - atmospheric absorption coefficient

a - constant of proportionality (slope of velocity gradient curve)

ΔA_1, ΔA_2 - cross-sectional area

c - phase velocity (propagation speed)

c_o - initial phase velocity

c_m - maximum phase velocity

D - thickness of layer

DS - change in distance along a ray path

DT - change in time

DX - change in horizontal range

DZ - height attained in a layer

F - some function

∇F - gradient of F

$\nabla \cdot F$ - divergence of F

$\nabla^2 F$ - Laplacian of F

ΔF - change in F

f - frequency

$f_{r,O}$ - relaxation frequency of oxygen

$f_{r,N}$ - relaxation frequency of Nitrogen

g - velocity gradient

g_i - velocity gradient in i^{th} layer

H_n - Heaviside function (derivatives and integrals)

h - per cent humidity

h_r - relative humidity

I - identity vector

i - unit intensity

j - $\sqrt{-1}$

k - constant = $\cos \theta_i / c_i$

L - intensity spreading loss

L_b - ground loss coefficient

m - angle between ray and wavefront normal

$\hat{n}$ - unit normal vector

n_b - number of ground reflections

P - total pressure

P_{sat} - saturation pressure

P_{so} - ambient pressure

p - acoustic pressure

p_i - components of ∇u

R - radius of curvature

r - integration variable

$\hat{r}$ - unit ray (normal to wavefront)

S - locates point on wave,constant in reference system of propagating wave

s - distance travelled

T_o - ambient temperature = 293.15 K (20^{o}C)

T_{01} - triple point isotherm temperature

TL - total acoustic loss

t - time

$u(\vec{x})$ - time for a wavefront to travel to $\vec{x}$

v - particle velocity

W_1, W_2 - wavefronts

x_c - horizontal distance travelled by the critical ray

$\vec{x}$ - vector at space coordinates, also horizontal components

z - vertical space component

z_m - height of maximum c

ε - range error

λ - wavelength

ϕ - potential velocity

ρ - acoustic density

ρ_o - ambient density

$\theta_n(\vec{x})$ - series coefficients in the expansion of ϕ

θ, θ_o, θ_i, $_m$ - angle ray makes from horizontal, initial, at height z_i, at height z_m

Ω - solid angle

ω - angular frequency

I. Introduction

The concepts presented in this paper are by no means new. It is hoped that the approach used will help to simplify and clarify a study that has been complicated by mathematical gymnastics. The theory presented is rigorous but the steps are logical and it is not assumed that the reader is already familiar with ray tracing.

Ray tracing is an approach that was developed in the field of optics. Geometrical optics, as it is called, has been used widely in different aspects of acoustics. It is most commonly used (in acoustics) in the specialties of fluid dynamics, shock theory[1], and non-linear acoustics[2] and called the method of characteristics. The methodology in these specialties is different than for sound propagation theory but the approach is very similar and the equations take the same form.

Ray tracing techniques have been used for many years in underwater sound propagation.[3] In recent years many acoustic approaches have been used in meteorology. Of these SODAR (SOund Detection And Ranging) has been used to determine acoustic rays and from resulting data to approximate temperature profiles under inversion conditions (increase of temperature with height).[4] The method of ray tracing has been promoted in the field of outdoor sound propagation partially due to new interest in noise control. In sound ranging applications the distance to the sound source is different than simply the product of sound speed and travel time in non-homogeneous media. Ray tracing is seen as a useful method in the study of propagation paths in non-homogeneous media where refraction is present.

A brief presentation will be given of the equations leading up to the wave equation. The eikonal equation will be derived assuming a series solution to the wave equation and taking the first terms of the expansion. Solution of the eikonal equation will be first done in the homogeneous (medium) case and then in the non-homogeneous case. Discussion then follows concerning caustics (high concentration of energy) and shadow zones (zones of silence).

A general discussion of the computer models will be presented and analysis of some data from field measurements will be analyzed and discussed. For more information concerning the use of the two computer programs see Appendix C. Appendix A contains the listing of an eigenray computer program which solves the eikonal equation for rays that start at a given source location and pass through a given receiver location. Appendix B contains a ray tracing routine which takes source location and starting angles either from the eigenray program or from some other source and plots the resulting rays.

The present program package has been designed to analyze ray paths over a flat terrain with specified vertical temperature and wind profiles. Attenuation because of spherical spreading and atmospheric absorption has been included.

Work is progressing to include ground effects and variable topography in the package. An eigenray routine designed for underwater use called CONGRATS (CONtinuous Gradient RAy Tracing System)[5] is being revised for atmospheric work. CONGRATS fits a continuous gradient to a discrete profile input. The final routine will also include the ability to change the temperature and wind profiles in a path.

II. Ray Tracing Theory

A. Derivation of the wave equation

For the sake of completeness the place to begin this study is with the basic equations leading up to the wave equation. The potential velocity ϕ is defined by

$$\vec{v} = \nabla \phi \tag{1}$$

The approach presented here is constructed around the potential velocity but it is noted that it can be developed around other quantities such as velocity or pressure equally as well.

The second equation needed is a statement of Newton's first law, that stress is equal to the negative of momentum flux. This is called Euler's equation and has the form

$$\nabla p = -\rho_o \, \partial v / \partial t$$

Substituting equation (1) here and after minor manipulation, Euler's equation for potential velocity becomes

$$p = -\rho_o \, \partial \phi / \partial t + \text{constant} \tag{2}$$

The state equation is a statement that pressure is a function of density If expanded in a series around the ambient density and only the first two terms are retained the linearized state equation becomes

$$p = P'(\rho_o)\ \rho$$

where ρ_o is the ambient density. $P'(\rho_o)$ is equal to the square of the propagation speed. So

$$p = c^2 \rho \tag{3}$$

is the linearized state equation to be used.

The final equation necessary to derive the wave equation is a statement of conservation of mass called the continuity equation. It states that the net flow of mass into (or out of) a volume is equal to the net change of mass in that volume and has the form

$$\nabla \cdot \vec{v} = -1/\rho_o \quad \partial \rho / \partial t$$

Substituting from equation (3) for ρ and from equation(1) for v, this equation becomes

$$\nabla^2 \phi = - 1/(\rho_o c^2) \quad \partial p / \partial t$$

And substituting for p from Euler's equation (2) the wave equation results

$$\nabla^2 \phi = 1/c^2 \quad \partial^2 \phi / \partial t^2 \tag{4}$$

B. Derivation of the eikonal equation

The eikonal equation is a transformation of the wave equation describing, instead of the wave itself, the propagation of wave surfaces or wavefronts. Rays may be considered as packages of acoustic energy travelling normal to the wavefronts. Wavefronts are the loci of points which undergo the same motion at a given instant.

Rays in this theory are somewhat equivalent to characteristics in the method of characteristics used in both non-linear acoustics and shock theory. The difference is that characteristics take the role in these other specialties as carriers of discontinuities.[1] The theories are very closely related. In section C a solution to the eikonal equation is developed using techniques typical of the method of characteristics.

To derive the eikonal equation we begin by defining the wavefront by the equation

$$S(\vec{x},t) = 0 \tag{5}$$

where $\vec{x} = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$. In this analysis S has the dimension of time and for a fixed point can be thought of as the difference between the time that has past and the time necessary for the wave defined by ϕ to reach that point. Therefore for

$$S(\vec{x},t) < 0$$

it is seen that

$$\phi(\vec{x},t) = 0$$

Since S is constant in reference to a location on the wave (e.g. the wavefront), ϕ can be expanded in a series (Taylor) of the form

$$\phi = \begin{cases} \sum_{n=0}^{\infty} \theta_n(\vec{x})\, S^n/n! & S>0 \\ 0 & S<0 \end{cases} \qquad (6)$$

It will be seen later that $\theta_o(\vec{x})$ represents the variation in magnitude of the wave, or the factor of spreading loss.

It is intended that the series solution will be substituted into the wave equation in order to obtain equations for S and θ_n. The series of equation (6) is chosen due to the property that when derivatives of ϕ are taken the derivatives of

$$H_n = S^n/n!$$

are simply H_{n-1}, ie.

$$H'_n(S) = H_{n-1}(S)$$

Since the wave equation has two derivatives the form of H_n for negative n must be considered. If equation (6) is rewritten as

$$\phi = \sum_{n=0}^{\infty} \theta_n(\vec{x})\, H_n(S) \qquad (7)$$

and $H_n(S)$ is defined as

$$H_n(S) = \begin{cases} S^n / n! & S>0 \\ 0 & S<0 \end{cases}$$

it is noted that $H_0(S)$ is simply the Heaviside function and the $H_n(S)$ are simply its integrals. Therefore, using generalized functions, $H_n(S)$ for negative n can be defined where

$$H_{-1}(S) = \delta(S)$$

and

$$H_{-2}(S) = \delta'(S)$$

etc.

Taking derivatives of equation (7) yields

$$\frac{\partial \phi}{\partial t} = \sum_{n=0}^{\infty} \theta_n(\vec{x})\, H_{n-1}(S)\, \frac{\partial S}{\partial t}$$

$$\frac{\partial^2 \phi}{\partial t^2} = \sum_{n=0}^{\infty} \theta_n(\vec{x}) \left\{ H_{n-2}(S) \left(\frac{\partial S}{\partial t}\right)^2 + H_{n-1}(S)\, \frac{\partial^2 S}{\partial t^2} \right\}$$

$$\nabla \phi = \sum_{n=0}^{\infty} (\nabla \theta_n\, H_n(S) + \theta_n\, H_{n-1}(S)\, \nabla S)$$

$$\nabla^2 \phi = \sum_{n=0}^{\infty} \nabla^2 \theta_n\, H_n(S) + 2\nabla\theta_n \cdot (H_{n-1}(S)\, \nabla S)$$
$$+ \theta_n\, H_{n-2}(S)\, (\nabla S)^2 + \theta_n\, H_{n-1}(S)\, \nabla^2 S$$

And therefore the wave equation becomes

$$\sum_{n=0}^{\infty} \left(\theta_n\, H_{n-2}(S) \left\{ (\nabla S)^2 - \frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 \right\}\right.$$
$$+ H_{n-1}(S) \left\{ \left[\nabla^2 S - \frac{1}{c^2}\frac{\partial^2 S}{\partial t^2} \right] \theta_n + 2\nabla\theta_n \cdot \nabla S \right\}$$
$$\left. + \nabla^2 \theta_n\, H_n(S) \right) = 0$$

Grouping like terms gives

$$\left\{(\nabla S)^2 - \frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2\right\} \Theta_0 H_{-2}$$

$$+ \left\{\left[(\nabla S)^2 - \frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2\right] \Theta_1 + 2\nabla\Theta_n \cdot \nabla S + \left[\nabla^2 S - \frac{1}{c^2}\frac{\partial^2 S}{\partial t^2}\right] \Theta_0\right\} H_{-1}$$

$$+ \ . \ . \ .$$

In general, the wave equation will be satisfied if the coefficients of H_{-2}, H_{-1} etc. are equal to zero. In this analysis only the first two coefficients are considered. Therefore

$$(\nabla S)^2 - \frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 = 0 \qquad (8)$$

and

$$2\nabla\Theta_n \cdot \nabla S + \left\{\nabla^2 S - \frac{1}{c^2}\frac{\partial^2 S}{\partial t^2}\right\} \Theta_0 = 0 \qquad (9)$$

Equation (8) is called the eikonal equation. Its solution leads directly to the concept of rays since it describes the motion of the surface $S(\vec{x},t) = 0$. Rays are defined as the path normal to the wave surface.

C. Solution to the eikonal equation

1. General Discussion

To get a general feel for rays and what the eikonal equation says, a perturbation approach is in order. First of all the unit normal to the surface S is given by

$$\hat{r} = \frac{-\nabla S}{|\nabla S|} \tag{10}$$

Considering an initial position of the wavefront depicted by

$$S(\vec{x}_0, t_0) = 0 \tag{11}$$

and slightly perturbing all of the variables of space and time the surface is then defined by

$$S(\vec{x}_0 + \hat{r}\, s,\ t_0 + \Delta t) = 0 \tag{11a}$$

Then a derivative may be approximated by a finite difference between equations (11a) and (11). The result is

$$\hat{r}\nabla S\ \Delta s + \frac{\partial S}{\partial t}\ \Delta t = 0$$

and the ray velocity (normal to the wavefront) is then given by

$$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{-\frac{\partial S}{\partial t}}{\hat{r}\nabla S}$$

Substituting for $\hat{r}$ from equation (10) yields

$$\frac{ds}{dt} = \frac{\partial S/\partial t}{|\nabla S|} \tag{12}$$

From the eikonal equation (8) we see that

$$ds/dt = \pm c$$

Therefore we can say that the eikonal equation in general says that a wavefront has a normal velocity or the ray has a velocity of magnitude $\pm c$. This may appear to be a trivial result but its importance is that this does correspond directly to the solution of the wave equation. It means that no matter what changes in direction a wavefront may undergo it will propagate, in isotropic media, at the characteristic phase velocity of the medium at its location.

2. Homogeneous, isotropic media

By homogeneous, it is meant that the propagation velocity c is constant with respect to location and time or equivalently that temperature is constant (isothermal condition) and there is no wind. Isotropic conditions imply that c is the same regardless of the direction of propagation. This is one of the simplest of cases. The solution shows the equivalence of the eikonal equation under conditions that will be demonstrated later to the wave equation. It is common to consider homogeneous, isotropic media when solving the wave equation but the power of the ray technique is seen best when these conditions are relaxed.

Specifying the wavefront S as

$$S(\vec{x},t) = t - u(\vec{x}) = 0 \tag{13}$$

it can be seen that $u(\vec{x})$ locates the wavefront at $\vec{x}$ for various times. Substituting this into equations (8) and (9) yields

$$(\nabla u(\vec{x}))^2 = 1/c^2 \tag{14}$$

and

$$2\nabla u(\vec{x})\cdot\nabla\theta_0 + \nabla^2 u\,\theta_0 = 0 \tag{15}$$

Solving equation (14) then gives a solution to the eikonal equation.

Consider now the change of some quantity along the ray, ie. the first derivative in terms of the distance s, d/ds. $\nabla u(\vec{x})$ is normal to the wavefront. Equation (14) says that $c\nabla u(\vec{x})$ is unity and therefore this represents the unit normal to the wavefront. Multiplying this quantity by the change along $\vec{x}$ once again yields d/ds ie.

$$\frac{d()}{ds} = c\nabla u \cdot \nabla() \qquad (16)$$

This equation says that the change along the path is equal to the change normal to the wavefront. The so called characteristic equations are all derived directly from equation (16). These are the derivatives with respect to s of $\vec{x}$, ∇u, and u. Therefore

$$\frac{d\vec{x}}{ds} = c\nabla u \qquad (17)$$

And since ∇u is constant from equation (14)

$$\frac{d\nabla u}{ds} = c\nabla u\ (\nabla\cdot(\nabla u)) = 0 \qquad (18)$$

and

$$\frac{du}{ds} = c(\nabla u)^2 = c/c^2 = \frac{1}{c} \qquad (19)$$

Since ∇u is normal to the wavefront, equation (17) shows that the rays are also normal which is how we initially defined rays. It is noted that in anisotropic media the rays are not necessarily orthogonal to the wavefront (see section II-C-9)[1]. Equation (18) says that ∇u is constant along the ray. It is concluded, therefore, that the rays are straight lines, ie. ∇u is constant and c is constant, therefore from equation (17), the path along the ray and the vectorial distance vary by a constant and the rays must be straight lines. Equation (19) integrates to

$$u = s/c \tag{20}$$

which means due to equation (13) that for any time $t > 0$ that the wave surface $t = u = s/c$ is at a distance ct along the ray, ie. $s = ct$. These equations together state that rays can be constructed by drawing straight lines from the initial wavefront. Figures 1 and 2 are examples of this construction showing a spherical source and a plane source, respectively.

3. Energy conservation and attenuation due to spreading

It is noted that equation (15) can be rewritten as

$$\nabla \cdot (\nabla u \, \theta_0^2) = 0 \tag{21}$$

This is in a divergence form which usually indicates the conservation of something. It is common to think of this as an equation showing the conservation of energy. If we consider a flow from the wavefront W_1 at time $u = 0$ to the wavefront W_2 at time $u = t$ as shown in figure 3 and integrate over the volume defined by a narrow tube between the wavefronts we will obtain the constant energy flux law and be able to find the attenuation due to the rays becoming less dense (ie. spreading loss). First we use the divergence theorem defined by[6]

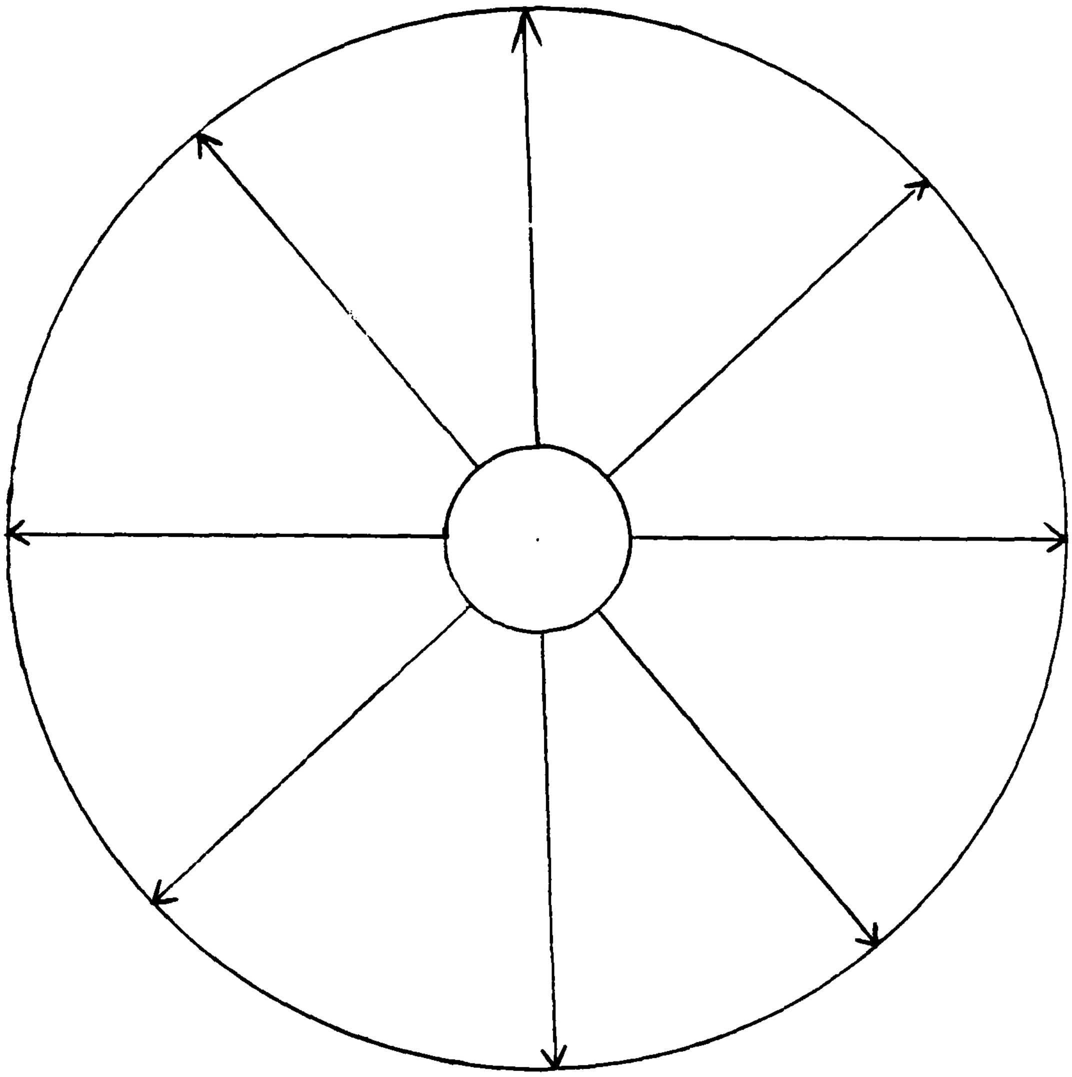

Figure 1 - Propagation from a spherical source in an homogeneous isotropic medium

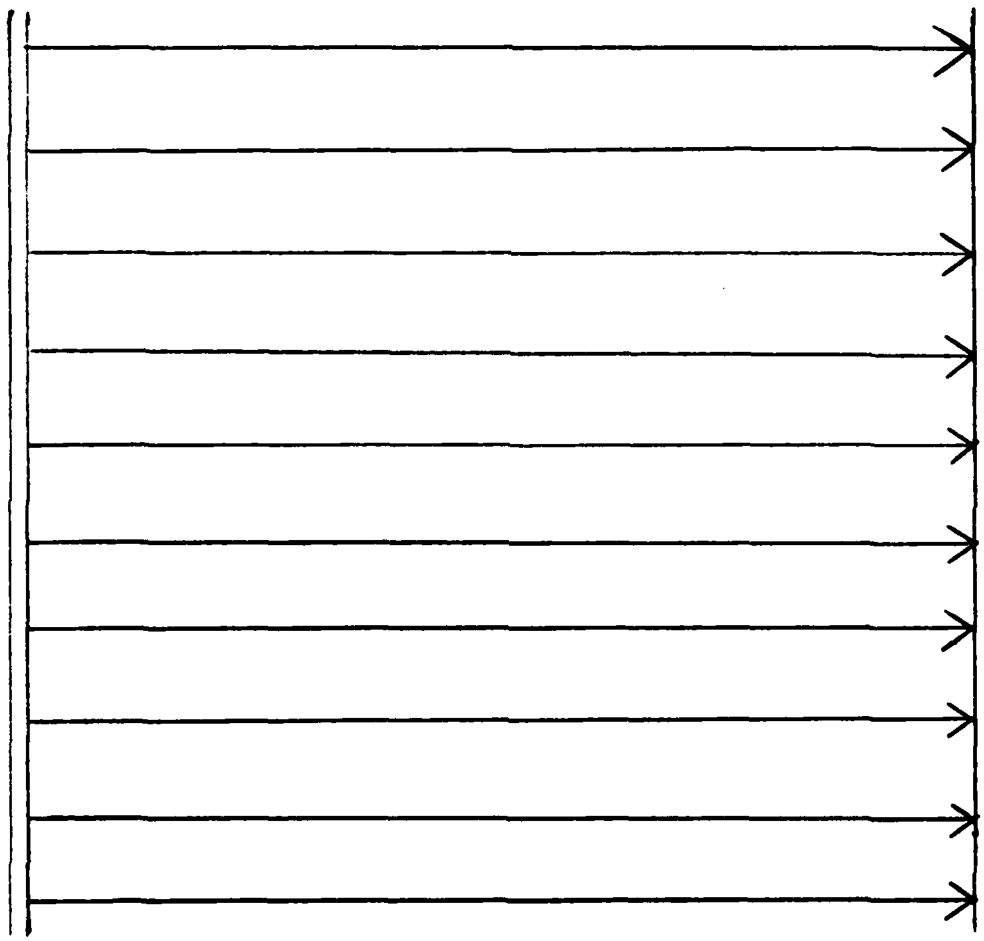

Figure 2 - Propagation from a plane source in an homogeneous isotropic medium

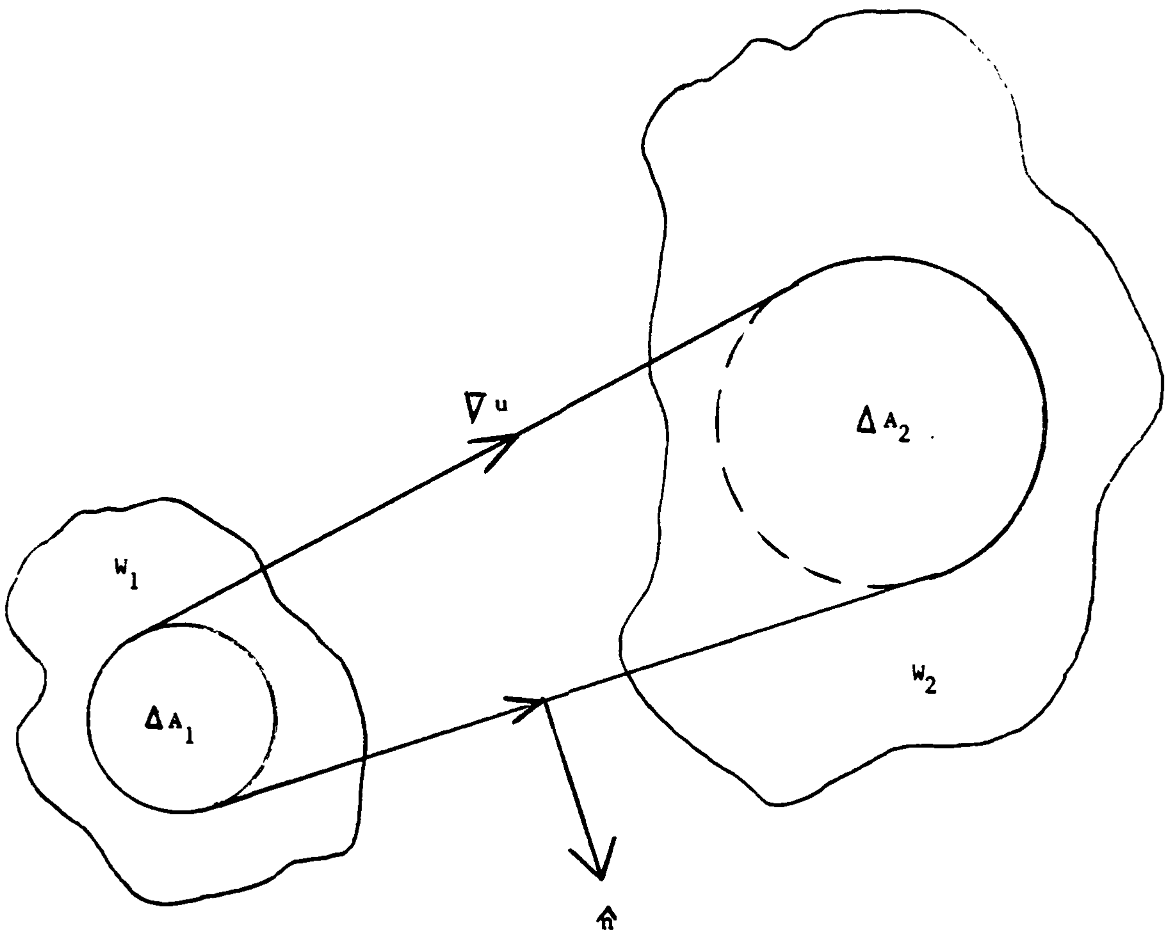

Figure 3 - Ray spreading

$$\iiint_V \nabla \cdot \vec{F} \, dV = \iint_S \vec{A} \cdot \hat{n} \, dS$$

on the volume integral of equation (21) and obtain

$$\iint_S (\nabla u \, \theta_0^2) \cdot \hat{n} = 0 \qquad (22)$$

On the sides of the narrow tube (see figure 3) $\hat{n}$ and $\vec{\nabla u}$ are orthogonal and therefore

$$\nabla u \cdot \hat{n} = 0 \qquad \text{on the sides.}$$

On W_2 and W_1, $\hat{n}$ and ∇u are in the same and opposite direction respectively, therefore

$$\begin{aligned} \nabla u \cdot \hat{n} &= |\nabla u| \qquad \text{on } W_2 \\ &= -|\nabla u| \qquad \text{on } W_1 \end{aligned}$$

Also from the eikonal equation (14)

$$|\nabla u| = 1/c$$

Therefore equation (22) is equivalent to

$$\iint_{W_2} \frac{1}{c} \theta_0^2 \, dS = 0 \qquad (23)$$

and

$$- \iint_{W_1} \frac{1}{c} \theta_0^2 \, dS = 0 \qquad (24)$$

and since c is constant in this case we may say

$$\iint_{W_1} \theta_0^2 \, dS = \iint_{W_2} \theta_0^2 \, dS$$

If we assume that we are integrating over a small package of rays with cross-sectional areas ΔA_1, and ΔA_2 on W_1 And W_2 respectively, the integrals may be approximated by

$$\theta_0^2(\vec{x}_1)\,\Delta A_1 = \theta_0^2(\vec{x}_2)\,\Delta A_2$$

This gives in the limit as ΔA_1 and ΔA_2 go to zero

$$\theta_0(\vec{x}_2)/\theta_0(\vec{x}_1) = (dA_1/dA_2)^{1/2} = (dA_2/dA_1)^{-1/2} \qquad (25)$$

The acoustic ray is the path of propagation of acoustic energy. Equation (25) means that divergence or convergence of rays indicates decreasing or increasing energy concentration, respectively. For example in plane waves

$$dA_2/dA_1 = 1$$

ie. the cross-sectional area of a bundle of rays stays constant along the propagation path and the rays are parallel. This indicates that

$$\theta_0 = \text{constant}$$

or that there is no spreading loss. For cylindrical and spherical rays

$$dA_2/dA_1 = R$$

$$dA_2/dA_1 = R^2$$

respectively. After substituting into equation (25) we have

$$\theta_0 \propto R^{-1/2} \qquad \text{for cylindrical rays}$$

and

$$\theta_0 \propto R^{-1} \qquad \text{for spherical rays.}$$

These terms are consistant with spreading losses associated with wave phenomena. In a non-homogeneous medium the losses will be similar. A ratio of sound speeds at one wavefront to the other will be included (ie. $c = c(x,y,z)$) but we may assume that the ratio will be near 1 and therefore can use the previous equations for θ_0 as the spreading factor. (See section III-A for a more precise spreading factor dependent on range rather than the distance travelled).

4. Prediction of Caustics

The examples presented in the last section for equation (25) concerned divergent rays and therefore showed examples of spreading loss. Another effect predicted by equation (25) is that of caustics. Caustics arise when an initial wavefront is concave away from the direction of propagation causing a focusing effect as in figure 4. The cusp shaped envelope is called a caustic. The region inside the envelope is triply covered by rays and energy is concentrated. On the caustic neighboring rays touch each other and therefore the bundle of rays described in the last section has a cross-sectional area of zero ie.

$$dA_2/dA_1 = 0$$

which predicts from equation (25) that

$$\theta_0 = \infty$$

Caustics or points in space where there is infinite acoustic energy are also predicted by the wave equation.[1] The question here is whether the linearized wave equation (4) applies in this case. It should be recognized that at caustics there is high acoustic energy concentration but because of non-linear effects it is not infinite.[7]

Caustics will also be evident in non-homogeneous and anisotropic media but are not as easily described as the cusp shaped envelope which arises in homogeneous, isotropic media.

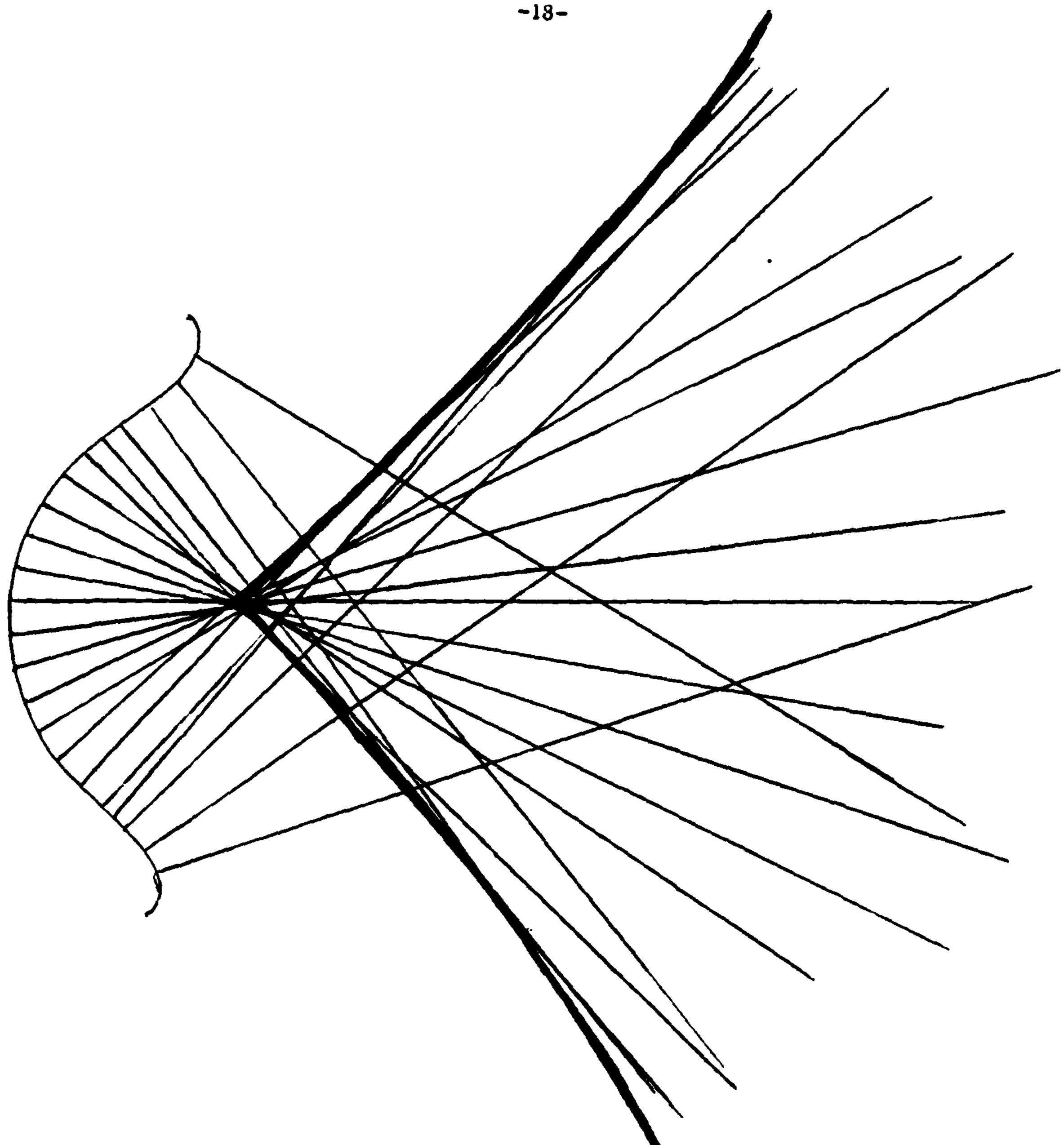

Figure 4 - Formation of a Caustic

5. Non-homogenous, isotropic, stratified media

By non-homogeneous it is meant that the phase velocity depends on location ie.

$$c = c(x,y,z)$$

In most cases c is considered as a function of height only, but for comparison the more general equations are presented and then the equations for the simpler stratified case.

The only difference in the eikonal equation (14) is that c is no longer constant. Using equation (16) will still give the proper characteristic equations for $d\vec{x}/ds$, $d\,\nabla u\,/ds$ and du/ds as

$$\frac{d\vec{x}}{ds} = c\nabla u \tag{26}$$

$$\frac{d\nabla u}{ds} = c\nabla u(\nabla\cdot\nabla u) = \frac{1}{2}\,c\nabla\cdot(\nabla u)^2$$

and because of the eikonal equation (14)

$$\frac{d\nabla u}{ds} = \frac{1}{2}\,c\nabla\cdot(c^{-2}) = \frac{-c}{c^3}\,\nabla c$$

therefore

$$\frac{d\nabla u}{ds} = -\,\frac{\nabla c}{c^2} \tag{27}$$

and

$$\frac{du}{ds} = c(\nabla u)^2 = \frac{c}{c^2} = \frac{1}{c} \tag{28}$$

Since ∇u is normal to the wavefront equation (26) like equation (17) says that the rays are also orthogonal to the wavefront.

However, equation (27) is different than equation (18) and says that $c\nabla u$ is no longer constant on a ray path and the combination of these two equations says that the rays bend around in response to the gradient of the phase velocity (∇c). The negative sign in equation (27) indicates that the rays bend toward a region of lower velocity. Solving equations (26) and (27) simultaneously, gives the rays and equation (28) gives the travel time by

$$u = \int_{\text{along ray}} \frac{ds}{c(\vec{x})} \tag{29}$$

In a stratified medium these equations simplify considerably so that they are more easily solved. In this case the phase velocity depends only on height ie.

$$c = c(z)$$

The characteristic equations become

$$\frac{d\vec{x}}{ds} = c\nabla_{x,y} u \qquad \frac{dz}{ds} = c\nabla_z u \tag{30}$$

$$\frac{d\nabla_{x,y} u}{ds} = 0 \qquad \frac{d\nabla_z u}{ds} = \frac{-\nabla_z c}{c^2} \tag{31}$$

$$\frac{du}{ds} = \frac{1}{c} \tag{32}$$

where $\vec{x}$ and $\nabla_{x,y}$ are the horizontal components and gradient respectively, and z and ∇_z are the verticle component and gradient. From equation (31) $\nabla_{x,y} u$ is constant and from equation (30) we see that $d\vec{x}/ds$ gives the angle from the horizontal that the ray makes as

$$\frac{d\vec{x}}{ds} = \cos\theta = c(z)\,\nabla_{x,y} u$$

If a subscript zero refers to an initial point we have

$$\nabla_{x,y} u = \frac{\cos\theta_0}{c_0} \tag{33}$$

and since $\nabla_{x,y} u$ is constant we can write the equation

$$\frac{\cos\theta}{\cos\theta_0} = \frac{c(z)}{c_0} \tag{34}$$

which is Snell's law in optics. From the eikonal equation we know that

$$(\nabla_{x,y} u)^2 + (\nabla_z u)^2 = (\nabla u)^2 = 1/c^2$$

and substituting from equation (33) gives

$$\nabla_z u = \left\{ \frac{1}{c^2(z)} - \frac{\cos\theta_0}{c_0} \right\}^{1/2} \tag{35}$$

to solve for rays the ray equations (30) may be combined into

$$\frac{d\vec{x}}{dz} = \frac{\nabla_{x,y} u}{\nabla_z u}$$

Substituting equations (33) and (35) and integrating yields the equation

$$\vec{x} - \vec{x}_0 = \int_{z_0}^{z} \frac{c(z)\cos\theta_0/c_0}{(1-c^2(z)\cos^2\theta_0/c_0^2)^{1/2}}\,dz \tag{36}$$

which describes a ray with initial angle θ_0 at a point (x_0, y_0, z_0). From equations (32) and (30) the ray travel time is given by

$$u = \int \frac{ds}{c} = \int_{z_0}^{z} \frac{dz}{c^2 \nabla u_z}$$

or

$$u = \int_{z_0}^{z} \frac{dz}{c(z)\,(1 - c^2(z) \cos^2 \theta_0 / c_0^2)^{1/2}} \qquad (37)$$

These last equations (36) and (37) are the basis of the model presented in this paper. In the next section the question of when the eikonal equation is valid is considered followed by a section which discusses two particular phase velocity distributions.

6. Conditions of validity of the eikonal equation

It is emphasized that the eikonal equation is only an approximation to the linearized wave equation. The word linearized is stressed so that one is aware that the wave equation itself is not always valid and certainly an approximation to it would not be valid under non-linear conditions. One of these conditions, that resulting in caustics, has already been noted in section II-C-4.

In this section an harmonic solution to the wave equation (4) is considered and substituted, resulting in the eikonal equation with some descrepancy. Making this error small is the condition sought, so that the eikonal equation will be a good approximation to the wave equation.

First the assumption is made that the solution is time harmonic only if the wave has reached the spatial coordinate specified. The wavefront S as defined in equation (13) is an appropriate time frame to consider. It is also assumed that the amplitude of the wave may vary in space due to variations in the medium. The solution is then of the form

$$\begin{aligned} \phi &= A(\vec{x})\,\exp(j\omega S(\vec{x},t)) \\ &= A(\vec{x})\,\exp(j\omega(t - u(\vec{x}))) \end{aligned}$$

Substituting into the wave equation (4) the resulting equation is

$$\nabla^2 A - \omega^2 A(\nabla u)^2 - j(2\omega\nabla A\cdot\nabla u + \omega A\nabla^2 u) = -\omega^2 A/c^2$$

Separating the real and imaginary parts yields

$$-\frac{1}{\omega^2 A}\nabla^2 A + (\nabla u)^2 - \frac{1}{c^2} = 0 \qquad (38)$$

and

$$\nabla^2 u + \frac{2}{A}\nabla A\cdot\nabla u = 0 \qquad (39)$$

For u to be a solution to the eikonal equation the first part of equation (38) must be zero. This will be so if the amplitude of oscillation A is constant or linear in which case the second spatial derivative of A would be zero; or if the frequency is infinite. In general neither of these assumptions can be made. The previous condition may be relaxed by making the first term in equation (38) much smaller than the second ie.

$$\frac{\nabla^2 A}{\omega^2 A} \ll (\nabla u)^2$$

Using the eikonal equation (14) this becomes

$$\frac{c^2 \nabla^2 A}{\omega^2 A} = \left(\frac{\lambda}{2\pi}\right)^2 \frac{\nabla^2 A}{A} \ll 1 \qquad (40)$$

for convenience the gradient of a function will be defined over the distance of one wavelength so that

$$\nabla F = \Delta F / \lambda \qquad (41)$$

This will transform equation (40) into

$$\lambda \frac{\Delta \nabla A}{A} \ll 1 \qquad (42)$$

If this condition is met, u is a solution to the eikonal equation. For u to also be a good approximation to the wave equation or rather for the ray solution to be a good approximation to the wave solution, equation (39) must also be satisfied or

$$\nabla^2 u = -2 \frac{\nabla A}{A} \cdot \nabla u = -2 \frac{\nabla A}{A} \frac{1}{c}$$

and from equation (42) this gives

$$- \lambda \Delta c \nabla^2 u \ll 1 \qquad (43)$$

Taking the gradient of the eikonal equation (14) we have

$$2(\nabla u) \nabla^2 u = -2 c^{-3} \nabla c$$

or using the square root of the eikonal equation

$$\nabla^2 u = - \frac{\nabla c}{c^2}$$

Substituting into equation (43) we have

$$\lambda \frac{\Delta \nabla c}{c} \ll 1$$

or from equation (41) and knowing that changes in c are small

$$\Delta \nabla c \ll \nabla c \qquad (44)$$

This is the condition sought. It states that a solution to the eikonal equation will be a good approximation to a solution of the wave equation if the change in the gradient of the phase velocity over a wavelength is small compared to the gradient itself. Therefore ray solutions are valid if there are no large changes or discontinuities in the phase velocity profile.

7. Formation of a shadow zone in a stratified medium

It was shown in section II-C-5 that rays bend toward areas of lower phase velocity. From this simple concept it can be surmised that if there exists a maximum phase velocity at some height z_m above a source that a shadow zone will be formed. A shadow zone is an area where no acoustic energy penetrates. More specifically, rays near the height z_m will bend either upward or downward away from that height. This is illustrated in figure 5. The limiting ray which defines the boundary of the shadow zone is that ray which becomes horizontal at height z_m. This ray is often called a split - beam ray since it may be bent upward or downward and theoretically is handled by considering that it goes both ways.

From Snell's law, equation (34) we have that

$$\theta = \cos^{-1}(c \cos \theta_0 / c_0) = \cos^{-1}(c \cos \theta_m / c_m) \qquad (45)$$

So that as c increases ie as the ray nears height z_m, θ will decrease. So that this equation is defined for all values of θ

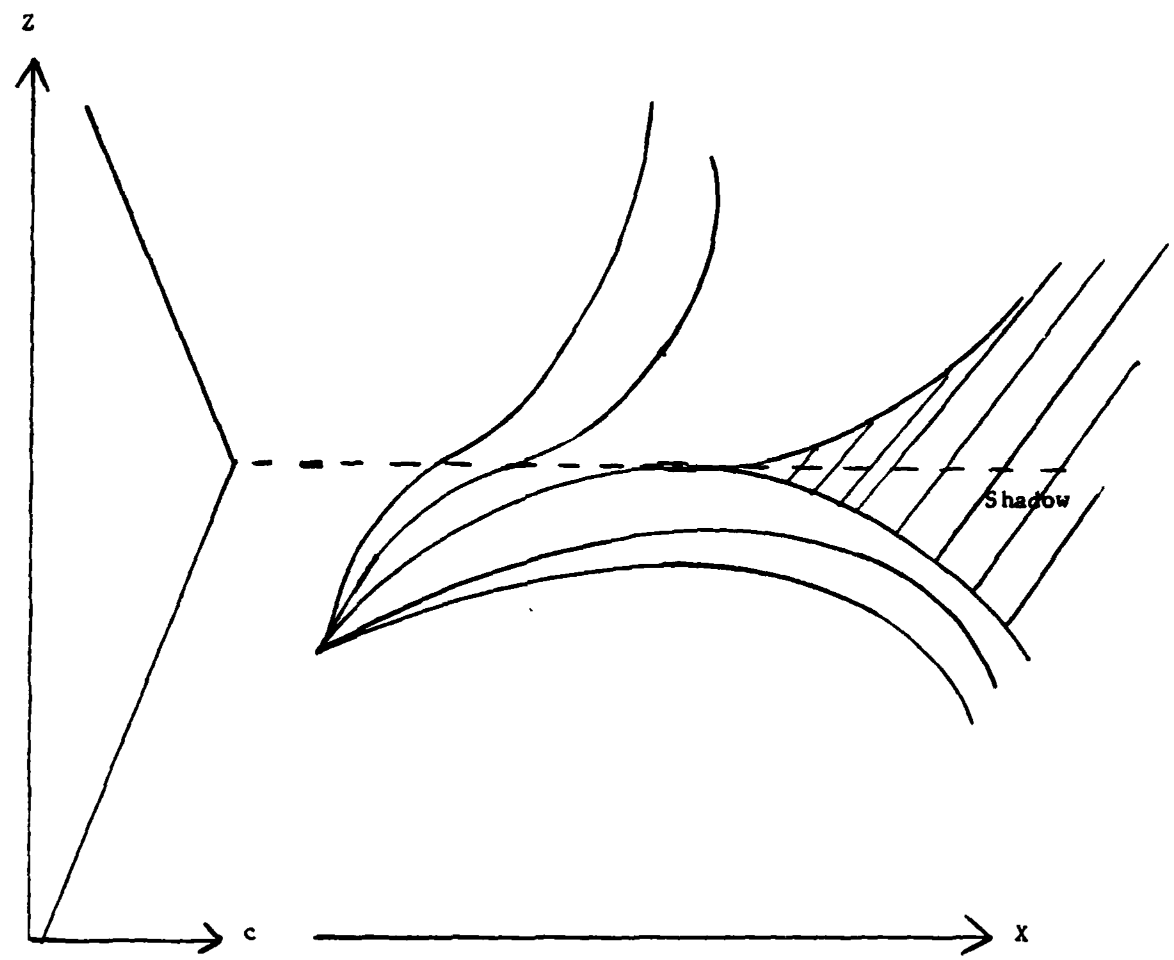

Figure 5 - Formation of a shadow zone

and c, c_m can be chosen to be the maximum phase velocity and θ_m the angle at the height associated with c_m.

In general, if the initial angle of a ray is zero, equation (45) says that the ray will not be horizontal again until it reaches a level with $c = c_0$. In figure 5, if the initial angle

$$\theta_0 > \cos^{-1} c_0/c_m \tag{46}$$

then θ_m will be greater than zero and the rays will penetrate the level of maximum sound speed and continue upward. However, if

$$\theta_0 < \cos^{-1} c_0/c_m \tag{47}$$

the cos θ increases to 1 and θ decreases to zero at the height of z defined by the equation

$$c(z) = c_0/\cos\theta_0 \tag{48}$$

At this point the ray bends downward.

The critical ray is the split - beam ray. This results when

$$\theta_m = 0$$

which occurs when

$$\theta_0 = \cos^{-1} (c_0/c_m) \tag{49}$$

A critical distance along the ground may be defined where the split - beam ray intersects the ground. It is instructive to use a case as in figure 5 where the velocity gradient is linear up to a maximum velocity. Equation (36) gives us the distance travelled in integral form Placing the sound source on the ground sets $z_0 = 0$ and we can define $x_0 = 0$. The phase velocity is then

$$c = c_0 + c_0 az$$

ie. there is a linear velocity gradient. Equation (36) then becomes

$$x = \int_0^z \frac{A(1 + az)\,dz}{(1 - A^2(A + az)^2)^{1/2}}$$

Where $A = \cos\theta_0$. Setting $r = 1 + az$ and $dr = adz$ gives

$$x = \int_1^{1+az} \frac{A\,r\,dr}{a(1 - A^2 r^2)^{1/2}}$$

$$= \frac{1}{aA}\left(-(1 - A^2 r^2)^{1/2}\right)\Bigg|_1^{1+az}$$

After slight manipulation this reduces to

$$\left\{x - \frac{1}{aA}(1 - A^2)^{1/2}\right\}^2 + \left\{z + \frac{1}{a}\right\}^2 = \frac{1}{a^2 A^2}$$

which is an equation for a circle. Substituting for A gives

$$\left\{x - \frac{\tan\theta_0}{a}\right\}^2 + \left\{z + \frac{1}{a}\right\}^2 = \frac{1}{a^2\cos^2\theta_0}$$

which is a circle centered at

$$(\tan\theta_0/a,\ -1/a)$$

with radius

$$R = 1/a\cos\theta_0$$

This means that for a linear velocity gradient rays will follow the arc of a circle.

For the distance travelled along the ground, $z = 0$ and

$$\left\{ x - \frac{\tan \theta_0}{a} \right\}^2 = \frac{1}{a^2 \cos^2 \theta_0} - \frac{1}{a^2}$$

$$= \frac{1}{a^2} \tan^2 \theta_0$$

Therefore the ray travels horizontally

$$x = \frac{2 \tan \theta_0}{a} \qquad (50)$$

before reaching the ground again. Using equation (49) for θ_0 defines this distance for the critical ray by

$$x_c = \frac{2 \tan (\cos^{-1}(c_0/c_m))}{a} \qquad (50a)$$

where a is the slope of the velocity profile.

Actually rays from the source may penetrate the shadow zone by multiple reflections off the ground. If in addition there exists a local maximum characteristic velocity above the maximum c_m, rays may again be bent downward into the shadow zone. For a more precise treatment one must include the effects of diffraction which are not readily defined using rays. However, the existence of shadow zones has been experimentally observed as low intensity zones.[3,8]

8. Waveguides

Waveguides result from the existence of a raised minimum velocity. This is illustrated in figure 6. The derivation of this result is similar to that for the shadow zone. If the initial angle is specified by equation (46) the ray will penetrate into the region of higher velocity. However if equation (47) describes the initial angle the cos θ will increase to 1 and θ will decrease to 0 and the ray will bend downward. At this point the ray crosses the minimum value of c again and will repeat the pattern symmetrically about the height of the minimum.

Waveguides are important in a discussion of ray theory since it allows a ray to propagate for a long distance without reflections and probable losses from ground interactions.

9. Anisotropic, homogeneous media

In anisotropic media the phase velocity is dependent on orientation. A simple example is when sound propagates in a wind. The sound speed will be greater in the direction of the wind than orthogonal to it. The eikonal equation (14) still remains in the same form, with the phase velocity now a vector, ie.

$$(\nabla u(\vec{x}))^2 = 1/(\vec{c})^2 \qquad (51)$$

In this case only homogeneous media are being considered so $\vec{c}$ is constant. The characteristic equations arise from equation (16)

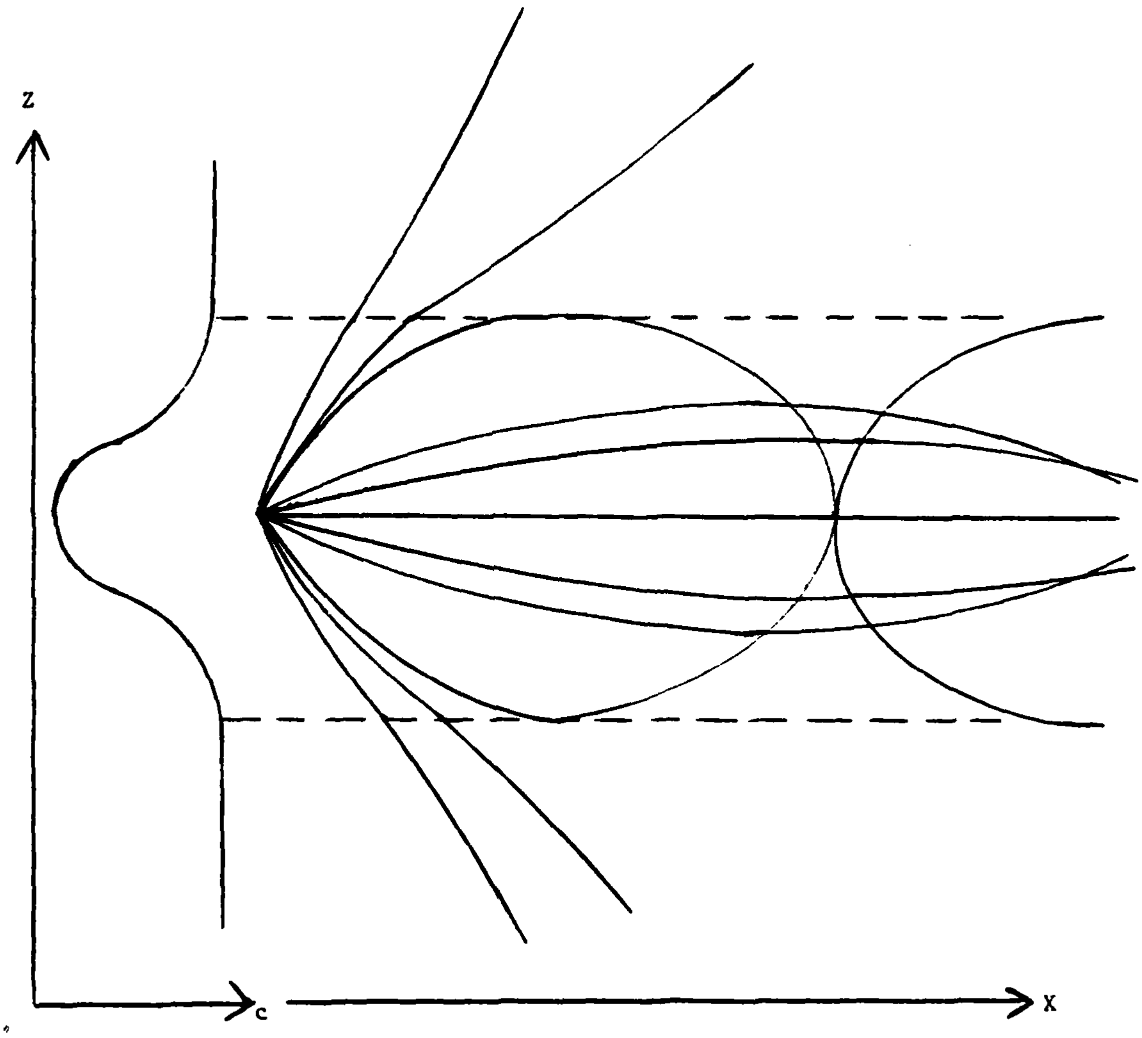

Figure 6 - Formation of a waveguide

in the following form;

$$\frac{d\vec{x}}{ds} = \vec{c}(\nabla u \cdot \vec{I}) \qquad (52)$$

$$\frac{d\nabla u}{ds} = 0 \qquad (53)$$

$$\frac{du}{ds} = \nabla u \cdot \frac{d\vec{x}}{ds} = \nabla u \cdot \vec{c}(\nabla u \cdot \vec{I}) \qquad (54)$$

Equation (53) says that the gradient of u is constant making the rays straight lines as would be assumed in a homogeneous medium. However, the ray direction specified by equation (52) will be parallel to the wavefront normal if and only if

$$\vec{c} \propto \nabla u$$

This will be true if and only if

$$(\vec{c} \cdot \nabla u)^2 = F(p_1^2 + p_2^2 + p_3^2) \qquad (55)$$

where the p_i's are the components of ∇u and F specifies some function. Equation (55) says that the rays are normal to the wavefront only in isotropic media.

Integrating equation (54) and substituting u = t locates the wavefront at successive times ie.

$$u = s\nabla u \cdot \vec{c}(\nabla u \cdot \vec{I})$$

or

$$s = \frac{t}{\nabla u \cdot \vec{c}(\nabla u \cdot \vec{I})} \qquad (56)$$

The vectorial distance is specif ed by integrating equation (52) as

$$\vec{x} = s\vec{c}(\nabla u \cdot \vec{I}) \qquad (57)$$

From equations (51) and (57) the unit vector in the direction of the ray is

$$\vec{c}(\nabla u \cdot \vec{I})$$

Equation (12) gives the velocity normal to the wavefront as

$$\frac{ds}{dt} = \frac{1}{(\nabla s)} = \frac{1}{|\nabla u|} = c$$

Therefore the unit vector normal to the wavefront is

$$c\nabla u$$

The angle m between the normal and the ray path can then be given by

$$\cos m = \vec{c}(\nabla u \cdot \vec{I}) \cdot (c\nabla u) \tag{58}$$

Using just the individual components of the vectors in this expression will give the angles in each plane. Using expressions (58) and (56) the distance along the ray may be specified by

$$s = \frac{ct}{\cos m} \tag{59}$$

This equation says that the wavefront moves along the ray with speed c/cos m which is greater than c.

III. Computer Model Description

A. Basic development of equations

In the previous sections ray - theory has been developed and discussed. This section is devoted toward techniques used in the development of computer programs from the equations derived. There are two types of programs to be described. These are 1) graphic ray tracing programs and 2) eigenray programs. In both types of programs the sound velocity profile must be specified.

Since the sound velocity as a function of height is not easily measured other related units must be measured. The sound velocity is directly proportional to the square root of absolute temperature as given by

$$c = 20.05\ (T)^{1/2}$$

where c is in meters per second and T is in degrees Kelvin (= degrees Celsius + 273.2). Since this refers to propagation relative to the medium we must include the wind velocity in this formulation so that the equation specifies propagation relative to the ground ie.

$$c = 20.05\ (T)^{1/2} + WV \tag{60}$$

The factors T and WV can be measured using thermistors and anemometers, respectively and the vectorial direction of the wind using a bi - vane. Therefore the phase velocity as a function of height may be specified.

In the development of the characteristic equations it was necessary to use the vertical phase velocity gradient given by dc/dz. In modeling techniques it is usual to use a linear difference approximation to derivatives, therefore

$$g_i = \frac{c_{i+1} - c_i}{z_{i+1} - z_i} \tag{61}$$

where g_i is the gradient. Using many segments for the gradient will approximate a smooth curve fairly well and therefore other difference forms (e.g. logarythmic) are not used. It is intended, however, that for a small number of values of T and WV, to include equations from meteorological theory to interpolate other values. The present model does not include these interpolation methods.

The assumption for the model is that instead of a simple stratified medium, the medium is divided into layers and each layer has a linear gradient. We can therefore use the equations developed earlier to derive equations for each layer and follow individual rays from layer to layer.

Three cases must be considered: 1) the isovelocity case, 2) variable velocity when the ray penetrates the layer and 3) variable velocity when the ray is refracted back towards its entry level. The isovelocity case is really simply the homogeneous case discussed in section II-C-2. In this case $g = 0$ and the rays are straight lines. If D is the thickners of the layer and θ_i is the angle of the ray upon entering the layer the change in the x distance will be defined by

$$DX = D \cot \theta_i \tag{62}$$

From equation (20) the travel time is given by

$$DT = \frac{DS}{c} = \frac{(DX^2 + D^2)^{1/2}}{c} \tag{63}$$

Horizontal rays in a homogeneous layer present a special case that will not leave the layer and will travel straight.

When the velocity changes with height and the ray penetrates the layer equation (36) may be used to find DX. In this case $c(z) = gz$. Letting $k = \cos\theta_i / c_i$ we have

$$DX = \int_{z_i}^{z_{i+1}} \frac{gz\,k}{(1 - (gzk)^2)^{1/2}}\,dz$$

$$= \frac{1}{gk}(1 - (gzk)^2)^{1/2}\Big|_{z_i}^{z_{i+1}}$$

$$= \frac{1}{gk}\sin\theta(z)\Big|_{z_i}^{z_{i+1}}$$

$$= \frac{1}{gk}(\sin\theta_{i+1} - \sin\theta_i) \qquad (64)$$

In this case the travel time is given by equation (37) as

$$DT = \int_{z_i}^{z_{i+1}} \frac{dz}{gz(1 - (gkz)^2)^{1/2}}$$

$$= -\frac{1}{g}\ln\left|\frac{1 + (1 - (gkz)^2)^{1/2}}{gkz}\right|_{z_i}^{z_{i+1}}$$

$$DT = -\frac{1}{g} \ln \left| \frac{1 + \sin \theta(z)}{\cos \theta(z)} \right|_{z_i}^{z_{i+1}}$$

$$DT = \frac{1}{2g} \ln \left| \frac{(1 + \sin \theta_i)(1 - \sin \theta_{i+1})}{(1 + \sin \theta_{i+1})(1 - \sin \theta_i)} \right| \qquad (65)$$

The third and final case is when a ray is bent around and returns in the direction it entered the layer. First, it is noted from equation (48) that if the ray becomes horizontal at a point where the phase velocity is given, the highest value of z is defined by

$$c(z) = gz = \frac{1}{K} = \frac{c_i}{\cos \theta_i}$$

Second, it was shown in section II-C-7 that rays travel in a circular path. Also, the ray may turn before reaching the edge of a layer. Therefore, since there is circular motion, the height attained in a layer is given by

$$DZ = \frac{1}{gk} (1 - \cos \theta_i) \qquad (66)$$

$$= \frac{z_i}{\cos \theta_i} - z_i$$

where θ_i and z_i are measured at the entrance to the layer. If this difference is greater than the thickness of the layer, the ray will not be bent around in that layer. If DZ in this equation is less than D then DX is defined by equation (50)

$$DX = \frac{2 \tan \theta_i}{a} \qquad (50)$$

where a is the slope of the gradient given by

$$g_i = c_i a$$

The time DT can then be found using equation (37) with limits of integration z_i and z_i + DZ and doubling the result since the ray must return to its entry height. Therefore,

$$DT = \frac{1}{g} \ln \left| \frac{1 - \sin \theta_i}{1 + \sin \theta_i} \right| \tag{67}$$

Equations (62), (63), (64), (65), (66), (50) and (67) form the basis of the computer models. The total horizontal distance and time the ray undergoes, x and t, are found by adding all the DX's and DT's, respectively. The actual distance the ray travels, s, is given by the sum of DS's where

$$DS = (D^2 + DX^2)^{1/2} \tag{68}$$

for the homogeneous case, or because the radius of curvature is defined by

$$\frac{ds}{d\theta} = R$$

and R was given in section II-C-7 as

$$R = \frac{1}{a \cos \theta_i} = g_i \frac{\cos \theta_i}{c_i}$$

therefore for the non-homogeneous case

$$DS = g_i \frac{\cos \theta_i}{c_i} (\theta_{i+1} - \theta_i) \tag{68a}$$

In the case of atmospheric sound propagation there are only reflections from the ground. Ground reflections are specular and handled by taking the negative of the angle of incidence.

The first type of program, graphic ray tracing, is constructed from these equations and includes the reflections. The remainder of this program consists of graphics techniques.

Input to the ray tracing program includes the temperature and wind profiles and the location and angle of the sound source. For rays travelling upward it is also necessary to include a maximum height that is to be considered. This height can sometimes be conveniently chosen just above a raised inversion, (velocity is greater at a greater height). Appendix B contains a graphics ray tracing model.

Eigenray routines find rays that travel from a source location to a specified receiver location. This is accomplished by searching a range of angles and using a bisection method to zero in on the angle at the source. The program follows many rays by the method used for ray tracing and internally varies only the source angle until a solution is found. Once this is completed the sound field at the receiver may be ascertained.

In the prediction of the sound field one must include the effects of absorption and spreading losses. To obtain the intensity spreading loss a solid angle Ω is defined with symmetry around the z-axis so that

$$d\Omega = 2\pi \cos \theta_0 \, d\theta_0$$

where the angles are specified in figure 7. The unit of intensity will be defined by the ratio of $d\Omega$ to the area dA swept out by the wave surface. From figure 7 this is

$$i = \frac{d\Omega}{dA} = \frac{2\pi \cos \theta_0 \, d\theta_0}{2\pi x \sin \theta_h \, dx}$$

$$= \frac{\cos \theta_0 \, d\theta_0}{x \sin \theta_h \, dx} \qquad (69)$$

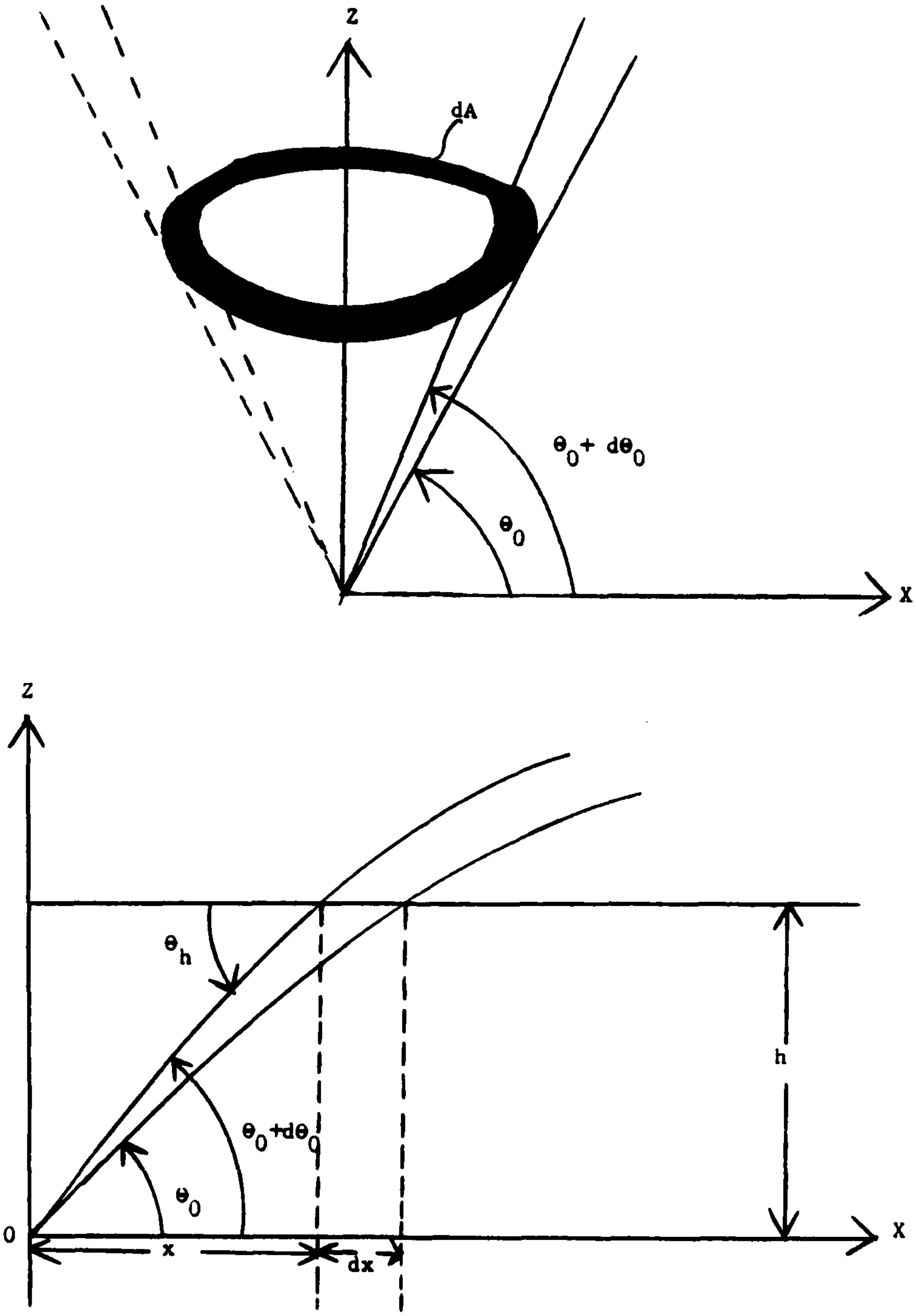

Figure 7 - Specification of solid angle and spreading

The horizontal range x is a function of the height h and initial angle θ_0 therefore, the horizontal unit of range is

$$dx = \frac{\partial x}{\partial \theta_0} d\theta_0$$

Substituting this into equation (69) and taking the reciprocal of the resulting function will yield the loss. On a log scale this is

$$L = 10 \log \frac{x \sin\theta_h \ \partial x/\partial\theta_0}{\cos \theta_0} \qquad (70)$$

Equation (64) is used to find an expression for $\partial x/\partial\theta_0$. It was said that x is the sum of the DX's, therefore

$$\frac{\partial x}{\partial \theta_0} = \frac{c_0 \sin \theta_0}{\cos^2 \theta_0} \sum_{i=0}^{n} \frac{\sin \theta_i - \sin \theta_{i+1}}{g_i}$$

$$+ \frac{c_0}{\cos \theta_0} \sum_{i=0}^{n} \frac{1}{g_i} \left(\cos \theta_i \frac{\partial \theta_i}{\partial \theta_0} - \cos \theta_{i+1} \frac{\partial \theta_{i+1}}{\partial \theta_0} \right)$$

$$= \frac{c_0 \sin \theta_0}{\cos^2 \theta_o} \sum_{i=0}^{n} \frac{1}{g_i} \left(\sin \theta_i - \sin \theta_{i+1} + \frac{\cos \theta_i \cos \theta_0}{\sin \theta_0} \frac{\partial \theta_i}{\partial \theta_0} \right.$$

$$\left. - \frac{\cos \theta_{i+1} \cos \theta_0}{\sin \theta_0} \frac{\partial \theta_{i+1}}{\partial \theta_0} \right) \qquad (71)$$

If we differentiate Snell's law, equation (34) we have

$$\frac{\partial \theta_i}{\partial \theta_0} = \frac{c_i}{c_0} \frac{\sin \theta_0}{\sin \theta_i} = \frac{\sin \theta_0 \cos \theta_i}{\cos \theta_0 \sin \theta_i}$$

Substitution into equation (71) and after slight manipulation we have

$$\frac{\partial x}{\partial \theta_0} = \frac{c_0 \sin \theta_0}{\cos^2 \theta_0} \sum_{i=0}^{n} \frac{1}{g_i}\left(\frac{1}{\sin \theta_i} - \frac{1}{\sin \theta_{i+1}}\right)$$

$$= - \frac{\sin \theta_0}{\cos \theta_0} \sum_{i=0}^{n} \frac{DX_i}{\sin \theta_i \ \sin \theta_{i+1}} \tag{72}$$

using equation (64).[10] Therefore the intensity spreading loss is

$$L = 10 \log \frac{x \sin \theta_{n+1} \sin \theta_0}{\cos^2 \theta_0} \sum_{i=0}^{n} \frac{DX_i}{\sin \theta_i \sin \theta_{i+1}} \tag{73}$$

To this value the ground absorption and atmospheric absorption must be added.

Presently ground losses are handled simply. The number of ground reflections n_b is counted and multiplied by a loss coefficient, L_b, provided by the user. It is intended to revise this by using a closed form where the impedance of the ground will be specified and phase information will be retained.

The atmospheric absorption coefficient is calculated using the American National Standard.[11] The necessary equations are included here for easy reference. The absorption coeffeicient is

$$\begin{aligned} \text{Alpha} = f^2 & (1.84 \text{ X } 10^{-11} (T/T_0)^{1/2} \\ & + (T/T_0)^{-5/2} (1.278 \text{ X } 10^{-2} (\exp(-2239.2/T)) \\ & /(f_{r,0} + (f^2/f_{r,0})) + 1.068 \text{ X } 10^{-1} (\exp(-3352/T)) \\ & /(f_{r,N} + (f^2/f_{r,N})))) \end{aligned} \tag{74}$$

in Nepers per meter. In this equation T is the temperature in degrees Kelvin and T_0 is the ambient temperature equal to 293.15 K;

f is the frequency of the source in Hertz and $f_{r,O}$ and $f_{r,N}$ are the relaxation frequencies in Hertz, for oxygen and nitrogen respectively , and are given by

$$f_{r,O} = (24 + 4.41 \times 10^{4} h \times ((0.05 + h)/(0.391 + h)))$$

and (75)

$$f_{r,N} = (T/T_0)^{-1/2}(9 + 350h \exp(-6.142((T/T_0)^{-1/3} - 1)))$$

In all of these equations the pressure is considered equal to the ambient pressure and so doesn't enter into the calculations. For the model the average value of temperature is used for T.

The variable h is the per cent humidity and can be calculated as

$$h = h_r(p_{sat}/p_{so}) \qquad (76)$$

where h_r is the relative humidity and the ratio of saturation pressure to ambient pressure can be calculated from

$$\begin{aligned} \log_{10}(p_{sat}/p_{so}) = {} & 10.79586\,(1 - (T_{01}/T)) \\ & - 5.02808 \log_{10}(T/T_{01}) \\ & + 1.50474 \times 10^{-4} \times (1 - 10^{-8.29692((T/T_{01})-1)}) \\ & + 0.42873 \times 10^{-3}\,(-1 + 10^{4.76955(1-(T_{01}/T))}) \\ & - 2.2195983 \qquad (77) \end{aligned}$$

where $T_{01} = 273.16$ is the triple point isotherm temperature.

The total loss is then given by

$$TL = 10 \log \frac{x \sin\theta_0 \sin\theta_{n+1}}{\cos^2\theta_0} \sum_{i=0}^{n} \frac{DX_i}{\sin\theta_i \sin\theta_{i+1}} \qquad (78)$$

$$+ \text{Alpha}\,(x) + n\,L_b$$

We now have the basis for an eigenray routine. Appendix A contains such a model. To graph the eigenrays, the output of the program in Appendix A is input into the program in Appendix B.

1. Eigenray routine improvements

Since the present model is for a horizontally homogeneous medium it can be surmized that after ground reflections and rays reach the initial height and angle the rays will follow the same pattern. Advantage is taken of this cyclic nature to speed up the calculation process. It is necessary to calculate only one cycle and compare the horizontal length of the cycle to the range.

Two types of intersection with the receiver are possible within one cycle; 1) as the ray is upward bound and 2) as the ray goes downward. A range of initial angles is swept through and rays coming near the receiver location are stored.

The rays then enter a ray convergence routine. The horizontal distance between where a ray intersects the receiver height and the receiver range is given by

$$\varepsilon = x - \text{Range} \tag{79}$$

A new ray is traced with the starting angle

$$\theta_0' = \theta_0 - \varepsilon/(\partial x/\partial\theta_0) \tag{80}$$

where $\partial x/\partial\theta_0$ is given by equation (72). This process will, under favorable conditions, reduce the value of ε , and is repeated until ε is smaller than a specified tolerance.

B. Some examples

The present models may be used to analyze a multitude of situations. Only a few can be discussed here.

First to be considered is a raised maximum phase velocity. It was shown in section II-C-7 that this would cause a shadow zone. The question discussed here is how intense must that maximum be to show a noticeable effect and also, what happens nearer the ground, below the maximum, since rays will be bent downward. In figure 8, there is an iso-velocity situation near the ground, and the velocity is maximum there. In the upper portion rays are bent upward, as would be expected, toward the lower velocity. The rays contained in the iso-velocity layer are straight and easily penetrate into the upper layer. Figure 9 shows a slight inversion in the lower level. The same rays are plotted here and the plot shows that the rays don't penetrate to the upper layer quite as easily as before. Rays are bent downward and trapped by the inversion. Figure 10 shows a more intense inversion. The rays are bent as before but to the right of the plot are more concentrated in the lower part. Figure 11 shows this concentration more clearly. More rays have been added between the rays in the lower portion of figure 10. It is noted in this figure that the upper section is much more concentrated than the lower, indicating a much higher intensity of sound. This point may be considered part of a caustic. It is easily seen from this set of figures that the more intense an inversion, the more rays may be trapped below. This would indicate that the sound intensity might likely be much higher in this region.

The ray tracing program may be used with a variable terrain as seen in figure 12. The eigenray routine is not yet capable of this. The problem is that the techniques used to speed up the computation time take advantage of the cyclic nature of rays which exists only if there are similar conditions over the entire terrain. Further investigation is necessary to allow for the ability to handle variable topography and maintain optimal use.

Table 1 shows the output of the eigenray routine for an inversion condition. This is a list of the rays

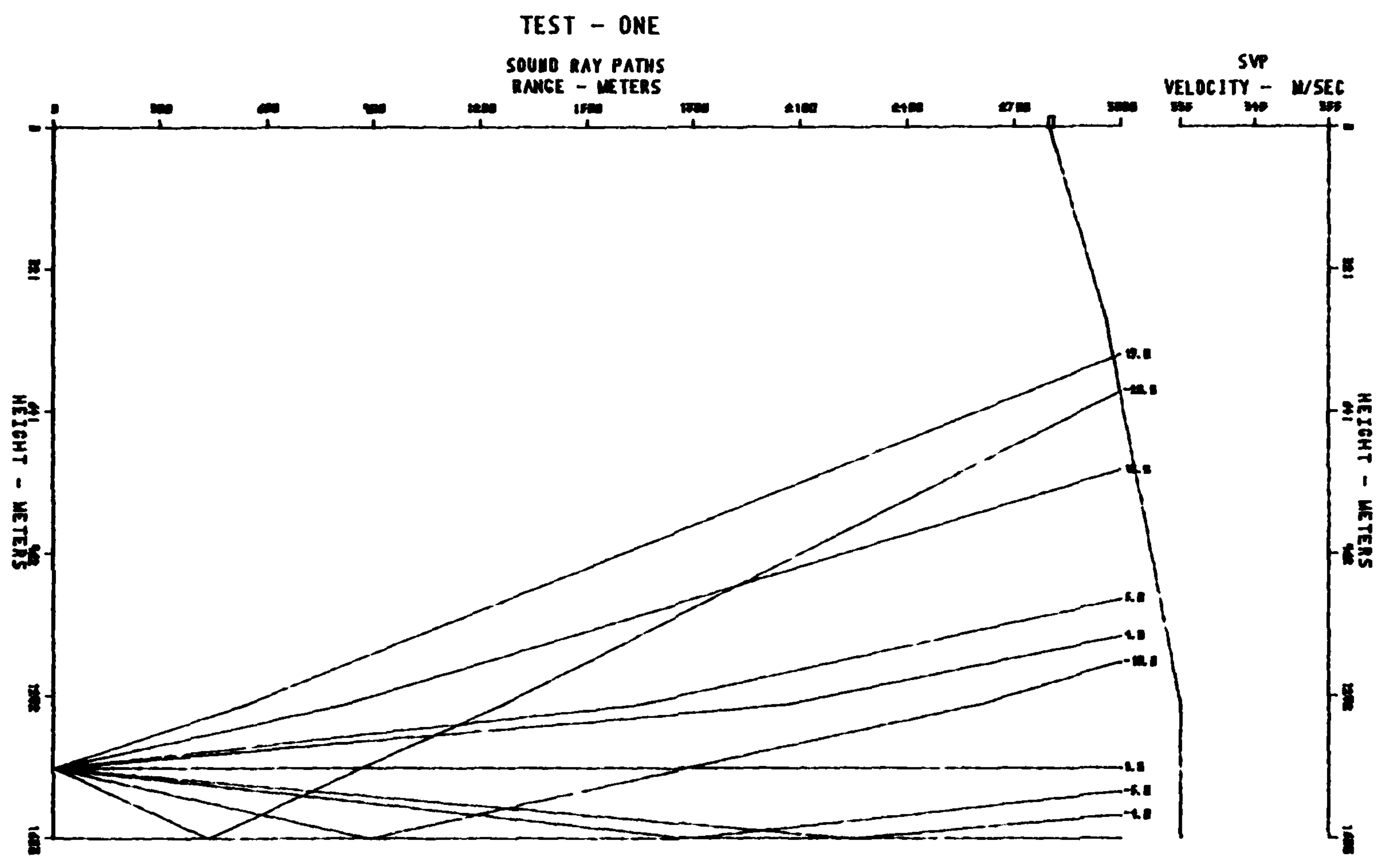

Figure 8 - Iso-velocity surface layer

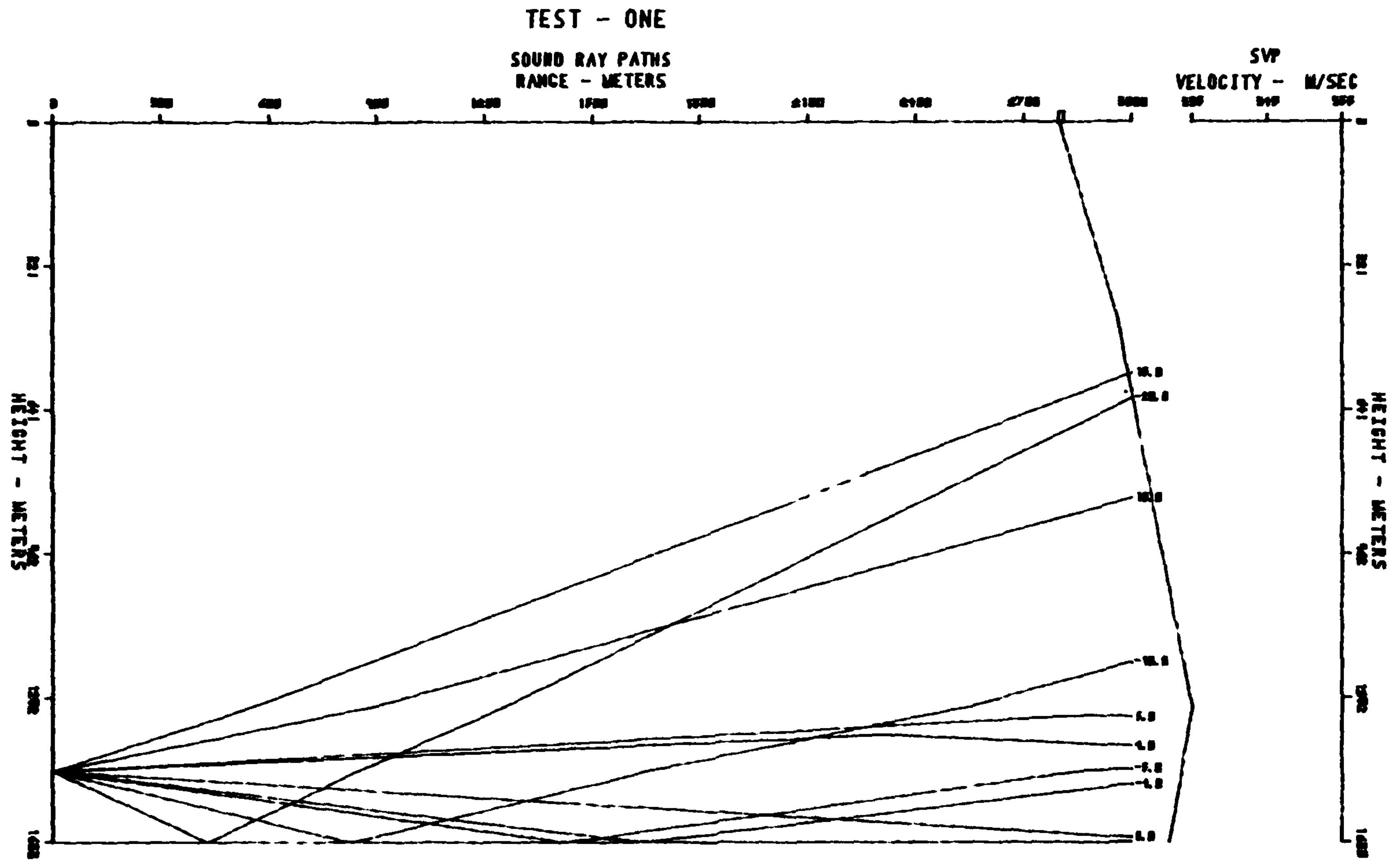

Figure 9 - Slight inversion in surface layer

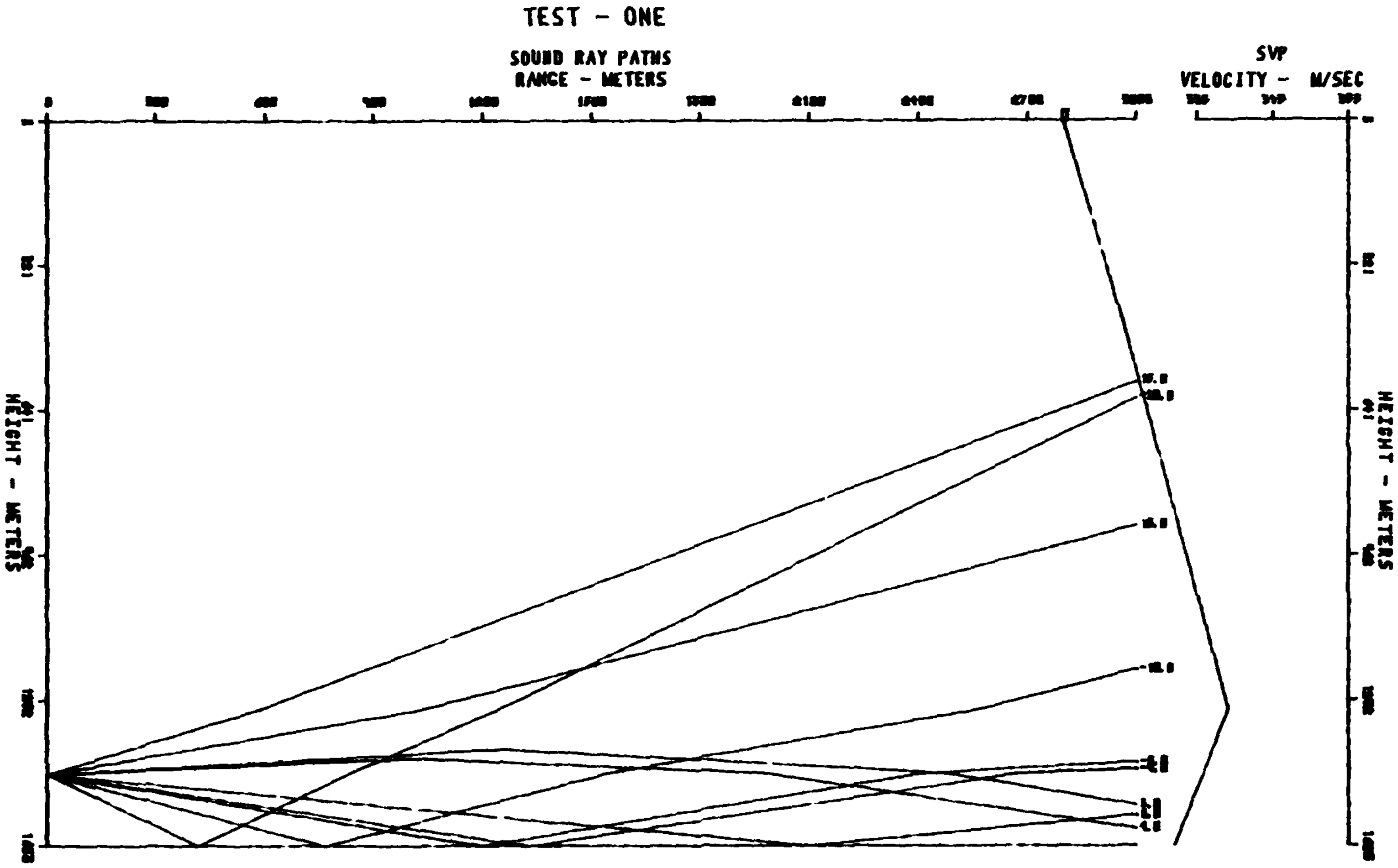

Figure 10 - Inversion in surface layer

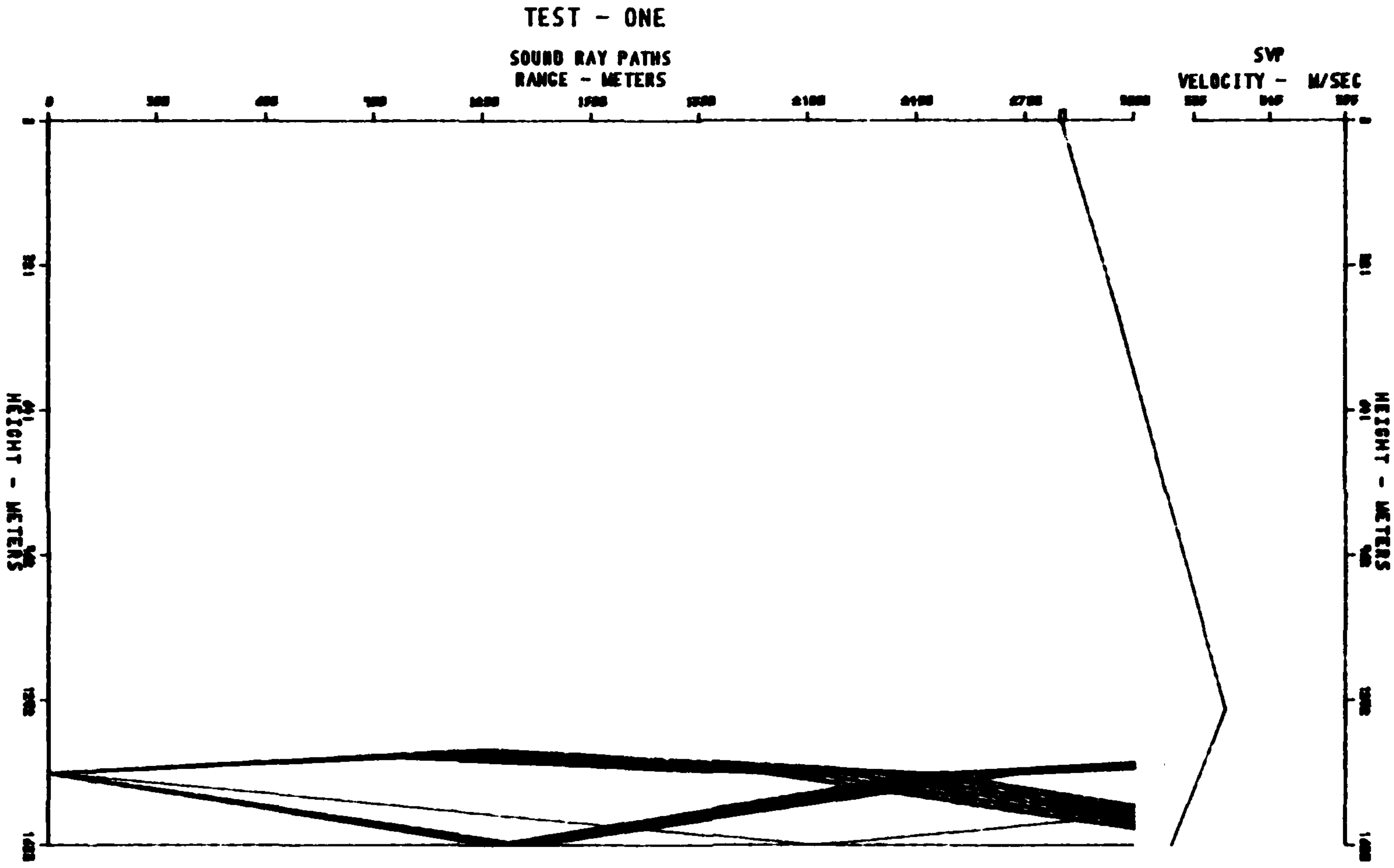

Figure 11 - Emphasized rays of figure 10

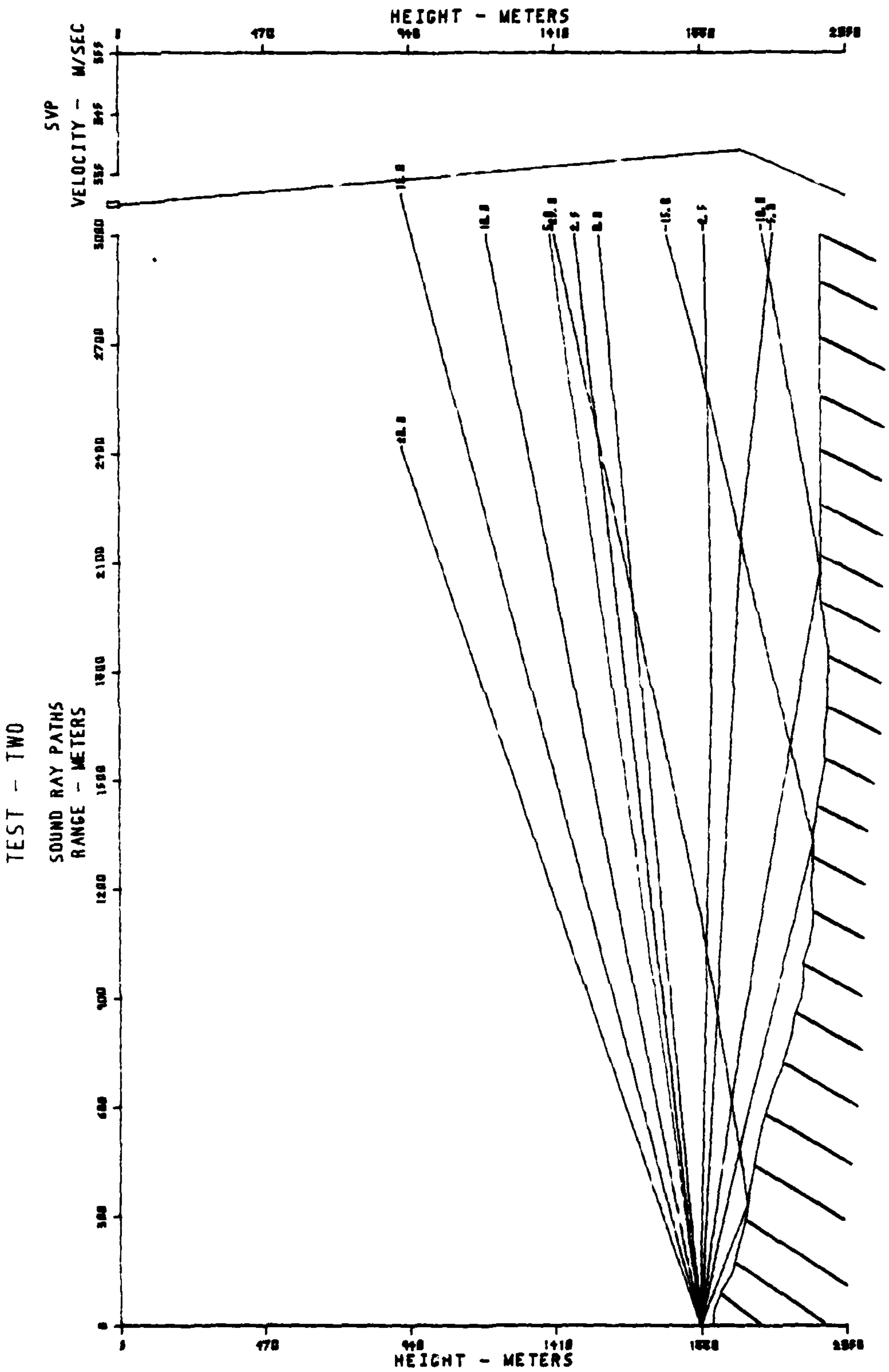

Figure 12 - Rays over a variable terrain

TABLE OF EIGENRAYS

TRAVEL TIME (SEC)	START ANGLE (DEG)	GROUND REFLECTIONS NO.	ANGLE	ATTN LOSS (DB)	SPREADING LOSS (DB)	TOTAL LOSS (DB)
0.0	5.117	2	9.120	5.38	68.19	73.57
0.0000	-5.117	2	9.120	5.38	68.19	73.57
0.0058	4.327	3	8.704	5.39	58.73	64.11
0.0058	-4.327	3	8.704	5.39	58.73	64.11
0.0074	2.807	4	8.061	5.38	58.55	63.93
0.0074	-2.807	4	8.061	5.38	58.55	63.93
0.0083	-1.996	5	7.817	5.38	57.72	63.10
0.0083	1.996	5	7.817	5.38	57.72	63.10
0.0090	1.406	6	7.688	5.38	56.50	61.89
0.0090	-1.406	6	7.688	5.38	56.50	61.89
0.0097	-0.916	7	7.614	5.38	54.42	59.80
0.0097	0.916	7	7.614	5.38	54.42	59.80
0.0102	0.391	8	7.569	5.38	48.52	53.90
0.0102	-0.391	8	7.569	5.38	48.52	53.90
0.0825	5.143	1	9.135	5.38	67.81	73.20
0.0940	4.520	2	8.801	5.39	59.12	64.51
0.1228	-0.585	8	7.582	5.38	50.26	55.64
0.1231	2.978	3	8.122	5.38	58.99	64.37
0.1463	2.171	4	7.863	5.38	58.35	63.73
0.1696	1.592	5	7.724	5.38	57.46	62.84
0.1952	1.135	6	7.644	5.38	56.15	61.53
0.2277	0.728	7	7.594	5.38	53.86	59.24
0.2370	-1.181	7	7.651	5.38	55.07	60.45
0.2835	0.266	8	7.564	5.38	47.29	52.67
0.3557	-1.800	6	7.770	5.38	56.91	62.29
0.5127	-2.620	5	7.998	5.38	58.01	63.40
0.7611	-4.117	4	8.602	5.39	58.25	63.63
1.2006	-5.095	3	9.108	5.39	68.62	74.00

Table 1 - List of eigenrays

that intersect the same receiver point specified as nine-hundred and fifteen meters. Figure 13 is a plot of a number of these rays (from the ray tracing program) and shows that in fact, they do intersect at the specified receiver location.

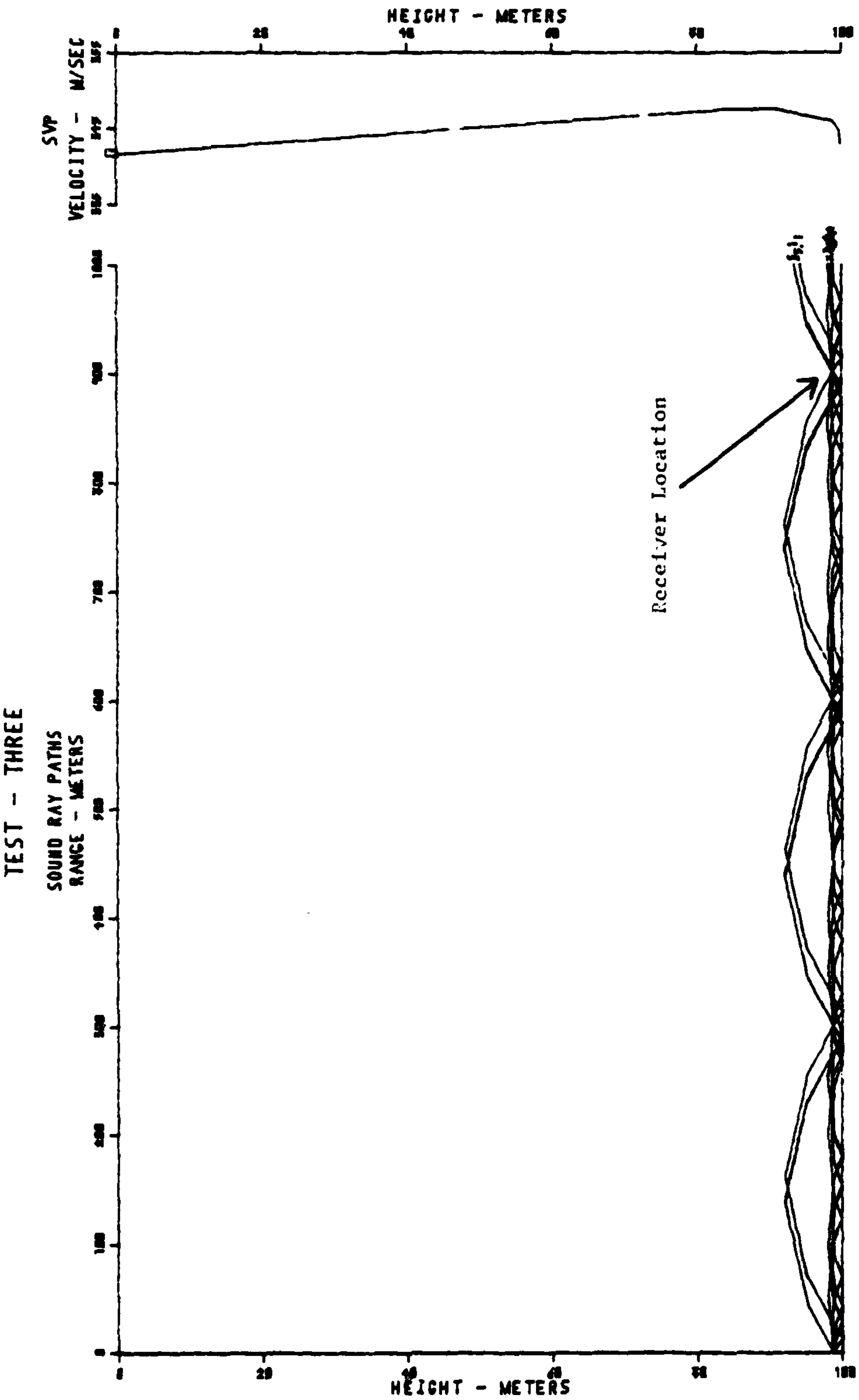

Figure 13 - Plot of eigenrays

IV. Summary

The purpose of this paper has been to discuss ray-tracing techniques. The equations have been derived from basic principles in a straight foward manner. Ray-tracing may be used in noise control applications as well as sound ranging. Ray solutions are good approximations to wave solutions under the condition that the velocity gradient doesn't change very much over a wavelength.

An analysis of the ray solution has been performed. Caustics are formed when rays are either bent toward each other or wavefronts have a concave profile. Linear theory predicts that there is infinite energy at a caustic. This is not so in reality due to non-linear effects. Caution must be taken when reviewing output from a ray analysis. Although the theory may predict infinite energy at a caustic, experiments show that the amount of energy may be very large, but not infinite.

Shadow zones occur when there exists an effective maximum sound velocity at some height. Waveguides occur when there is a raised minimum sound velocity.

In anisotropic media rays are not orthogonal to wavefronts. For the present models only isotropic media are considered. An understanding of how rays travel in anisotropic media is enlightening to real situations.

Computer programs have been developed to demonstrate ray techniques and are contained in the appendices of this paper. These programs have been used to show some examples.

The programs are presently being utilized in much research at the Noise Control Lab of The Pennsylavnia State University. They are being constantly revised for various uses.

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<u>Appendix A</u>

```
C      RAY PATH CALCULATION - MAIN PROGRAM
       COMMON /SX/DEP(100),VEL(100),GRAD(99),TEMP(100),WV(100)
       COMMON /R/TT(99),DB(99),ATN(99),ANGO(99),ANGS(99),ANGB(99)
       COMMON /R/NS(99),NB(99)
       COMMON /P/TLOSS(99)
       INTEGER TITLE
       DIMENSION TITLE(10),BL(10)
     5 READ(5,301,END=6) TITLE
       READ(5,302) NDFT,NVFT,NTF,NWV,NRFT,IVEL,ITEMP,IWV,IRHM
       IF ((IVEL+ITEMP).EQ.0) STOP
C IF SVP DATA IS TO BE INTERNALLY GENERATED, REPLACE 'STOP' BY
C APPROPRIATE 'GO TO' TO GENERATING ROUTINE.
       WRITE(6,305) TITLE
       NP=0
    10 NP=NP+1
       READ(5,303)DEP(NP),TEMP(NP),VEL(NP),WV(NP),NOMO
       IF (NOMO.EQ.0) GO TO 10
    15 CALL SSP(NP,NDFT,NVFT,NTF,NWV,IVEL,ITEMP,IWV)
       READ(5,400) NBL,POR,(BL(I),I=1,NBL)
       READ(5,304) TWIN
    20 READ(5,304) SD,TD,RANGE,ANGMAX,ANGMIN,FREQ,RHM
C      IF DEPTH IN FEET
       IF(NDFT.EQ.1) GO TO 61
C      CONVERT TO FEET
       SD=SD*3.28
       TD=TD*3.28
 61    IF (RANGE.EQ.0) GO TO 5
C       IS RANGE IN FEET
       IF(NRFT.EQ.1) GO TO 22
C      CONVERT METERS TO KILOYARDS
       RANGE=RANGE*1.09333/1000.
       GO TO 23
C      CONVERT FEET TO KYARDS
 22    RANGE=RANGE/3000.
 23    IF(IRHM.EQ.1) GO TO 21
C      DEFAULT VALUE OF RELATIVE HUMIDITY
       RHM=50.
 21    IF(ITEMP.EQ.0) GO TO 24
C      FIND AVERAGE TEMPERATURE IN DEGREES C
       AVTP=0.0
       IF(NTF.EQ.0) GO TO 26
       DO 27 I=1,NP
 27    AVTP=(TEMP(I)-32.)*5./9.+AVTP
       GO TO 28
 26    DO 29 I=1,NP
 29    AVTP=TEMP(I)+AVTP
 28    AVTP=AVTP/NP
       GO TO 31
C     DEFAULT AVERAGE TEMP 20 DEGREES C
 24    AVTP=20.0
```

```
 31    CALL RAY(NP,SD,TD,RANGE,ANGMAX,ANGMIN,FREQ,NRAY,RHM,AVTP)
       IF(NRAY.EQ.0) GO TO 71
       CALL TTORD(NRAY)
       TO=TT(1)
       DO 25 I=1,NRAY
   25  TT(I)=TT(I)-TO
       SS=20.*ALOG10(1E3*RANGE)
C      IF OUTPUT IN MKS OR BES
       IF(NVFT.EQ.1) GO TO 55
C      CONVERT TO MKS
       SD=SD*.304878
       TD=TD*.304878
       RANGE=RANGE*.9146341
       WRITE(6,356) TITLE,SD,TD,RANGE,FREQ,ANGMAX,ANGMIN,SS,TO
       GO TO 56
 55    WRITE(6,353) TITLE,SD,TD,RANGE,FREQ,ANGMAX,ANGMIN,SS,TO
 56    DO 45 K=1,NBL
       DO 30 I=1,NRAY
       TLOSS(I)=DB(I)
       IF (NB(I).EQ.0) GO TO 30
       XNB=NB(I)
       TLOSS(I)=DB(I)+XNB*BLOS(FREQ,POR,ANGB(I))+XNB*BL(K)+ATN(I)
   30  CONTINUE
       WRITE(6,450) BL(K)
       WRITE(6,354)
       WRITE(6,355) (TT(I),ANGO(I),NB(I),ANGB(I),
     1 ATN(I),DB(I),TLOSS(I),I=1,NRAY)
       CALL INTOUT(NRAY,TWIN,XIOM)
   45  CONTINUE
 71    IF(NRAY.EQ.0) WRITE(6,358)
    6  STOP
  400  FORMAT(I1,4X,11F5.1)
  450  FORMAT(15H BOTTOM LOSS = ,F5.1,//)
  301  FORMAT(10A4)
  302  FORMAT(9I1)
  303  FORMAT(4F10.4,1X,I1)
  304  FORMAT(7F10.3)
  305  FORMAT(1H1,10A4//)
  353  FORMAT(1H1,10A4//
     1   1H ,12HSOURCE DEPTH,F8.3,3H FT/
     2   1H ,12HTARGET DEPTH,F8.3,3H FT/
     3   1H ,5HRANGE,F8.3,5H KYDS//
     4   1H ,4HFREQ,F7.3,4H KHZ/
     5   1H ,9HMAX ANGLE,F6.1,4H DEG/
     6   1H ,9HMIN ANGLE,F6.1,4H DEG//
     7   1H ,12HSPH SPP LOSS,F7.2,3H DB/
     8   1H ,16H1ST ARRIVAL TIME,F8.3,5H SECS//)
  354  FORMAT(///,16X,18HTABLE OF EIGENRAYS//
     1   1H ,20HTRAVEL    START        ,
     2   35HGROUND       ATTN SPREADING   TOTAL/
```

```
     3  1H ,18H TIME    ANGLE    ,
     4  36HREFLECTIONS    LOSS   LOSS      LOSS/
     5  1H ,19H (SEC)   (DEG)     ,
     6  35HNO. ANGLE     (DB)   (DB)      (DB)//)
 355 FORMAT(1H ,F6.4,F9.3,2X,I4,F8.3,2F8.2,F10.2/)
356  FORMAT(1H1,10A4//
     1  1H ,12HSOURCE DEPTH,F8.3,3H M /
     2  1H ,12HTARGET DEPTH,F8.3,3H M /
     3  1H ,5HRANGE,F8.3,5H  KM //
     4  1H ,4HFREQ,F7.3,4H KHZ/
     5  1H ,9HMAX ANGLE,F6.1,4H DEG/
     6  1H ,9HMIN ANGLE,F6.1,4H DEG//
     7  1H ,12HSPH SPP LOSS,F7.2,3H DB/
     8  1H ,16H1ST ARRIVAL TIME,F8.3,5H SECS//)
358  FORMAT(10X,'NO RAYS FOUND')
     END
     SUBROUTINE RAY(NP,SD,TD,RANGE,ANGMAX,ANGMIN,FREQ,NRAY,RHM,AVTP)
C PROGRAM FINDS EIGENRAYS AND CALCULATES TRANSMISSION LOSS
C    NP = NUMBER OF POINTS IN SOUND SPEED PROFILE
C    SD = SOURCE DEPTH (FT)
C    TD = TARGET DEPTH
C    RANGE = SOURCE- TARGET HORIZONTAL RANGE (KYDS)
C    ANGMAX = MAX ANGLE SEARCHED (DEG)
C    ANGMIN = MINUMUN ANGLE SEARCHED
C    NRAY = NUMBER OF EIGENRAYS FOUND
C    RHM = RELATIVE HUMIDITY N PER CENT
C    AVTP = AVERAGE TEMPERATURE N DEGREES CELSIUS
C    AUX PRINT-OUT: SW7 ON - RAY SEARCH INFO
C                   SW8 ON - DF-BUG
      COMMON /SX/D1(100),V1(100),G1(99),T11(100),WV(100)
      COMMON /R/TT(99),DB(99),ATN(99),ANO(99),ANS(99),ANB(99)
      COMMON /R/LS(99),LB(99)
      DIMENSION D(102),V(102),G(101)
      DIMENSIONDD(2),ND(2)
      DOUBLE PRECISION PID,VKD,CVD,THOD,SITHD,CSTHD,SITH2D,CSTH2D
      DOUBLE PRECISION XD,DXD,XTD,RYARDD,SUMD,DSUMD
      IPDB=2
      IPRINT=2
      PID=3.14159265358979D0
      PI=SNGL(PID)
C MAX NUMBER OF RAYS (SIZE OF /R/ ARRAYS)
      NRAYMX=99
C     CALCULATE ATTN COEFF BY AMERICAN NATIONAL STANDARD
C     CHANGE TO DEGREES KELVIN
      AVTP=AVTP+273.15
C     CHANGE TO HZ
      FTT=FREQ*1000.
      T0=293.15
      T01=273.16
      PLR=10.79586*(1.-(T01/AVTP))-5.02808*ALOG10(AVTP/T01)+1.50474*10.*
```

```
     1*(-4.)*(1.-10.**(-8.29692*((AVTP/T01)-1.)))+0.42873*10.**(-3.)*(-1
     1.+10.**(4.76955*(1.-(T01/AVTP))))-2.2195983
      HM=RHM*10.**PLR
      FRO=24.+4.41*10.**4*HM*(0.05+HM)/(0.391+HM)
      FRN=(T0/AVTP)**.5*(9.+350.*HM*EXP(-6.142*((AVTP/T0)**(-1./3.)-1.))
     1)
C     ALPHA IN NEPERS/METER
      ALPHA=FTT**2*(1.84*10.**(-11)*(AVTP/T0)**.5+(AVTP/T0)**(-5./2.)*(1
     1.278*10.**(-2)*EXP(-2239.1/AVTP)/(FRO+FTT*FTT/FRO)+.1068*EXP(-3352
     1./AVTP)/(FRN+FTT*FTT/FRN)))
C     CONVERT TO DB/KYD
      ALPHA=ALPHA*868.589*3.048037*3.
C FIT SOURCE AND TARGET INTO SVP
      DO 5 J=1,NP
      D(J)=D1(J)
      V(J)=V1(J)
      IF(J.EQ.NP) GO TO 6
    5 G(J)=G1(J)
    6 LP=NP
      I=1
      IF (SD-TD) 10,11,12
   10 DD(1)=SD
      DD(2)=TD
      J=1
      GO TO 15
   11 DD(2)=SD
      J=2
      GO TO 15
   12 DD(1)=TD
      DD(2)=SD
      J=1
   15 IF (DD(J)-D(I)) 20,23,24
   20 LP=LP+1
      IP=LP-I
      DO 21 K=1,IP
      L=LP-K
      M=L+1
      D(M)=D(L)
      V(M)=V(L)
      IF(L.EQ.1) GO TO 26
      M=L-1
   21 G(L)=G(M)
   26 D(I)=DD(J)
      M=I-1
      V(I)=V(M)+G(M)*(D(I)-D(M))
      ND(J)=I
   22 IF (J.GE.2) GO TO 35
      J=2
      GO TO 15
   23 ND(J)=I
```

```
      GO TO 22
   24 IF (I.GE.LP) GO TO 30
      I=I+1
      GO TO 15
   30 ND(J)=LP
   35 IF (SD-TD) 40,41,42
   40 NSD=ND(1)
      NTD=ND(2)
      GO TO 60
   41 NSD=ND(2)
      NTD=NSD
      GO TO 60
   42 NTD=ND(1)
      NSD=ND(2)
C INITIALIZE RAY TRACE
   60 ANGO=ANGMAX
      RYARD=1E3*RANGE
      RYARDD=DBLE(RYARD)
      ERRMX=1.
      STEP=0.05
      NSTEP=0
      IRAY=0
      JRAY=0
      NRAY=0
      IJ=0
C IPRINT=1 IF  SS7 ON:  IPRINT=2  IF SS7 OFF
      IF (IPRINT.EQ.2) GO TO 65
      WRITE(6,802) SD,TD,RANGE,ANGMAX,ANGMIN,FREQ,ALPHA
      WRITE(6,801)
      IP=LP-1
      WRITE(6,800) (I,D(I),V(I),G(I),I=1,IP)
      WRITE(6,800) LP,D(LP),V(LP)
      WRITE(6,950)
C START NEW RAY
   65 K=NSD
C CHECK IF INITIAL RAY AT HIGHEST LIMIT
      IF (K.GT.1) GO TO 70
C DOES INITIAL 'LIMIT' RAY GO DOWNWARD ?
      IF (ANGO.GT.0.) GO TO 205
      IF ((ANGO.EQ.0.).AND.(G(1).GE.0.)) GO TO 205
C CHECK IF INITIAL RAY ON GROUND
   70 IF (K.LT.LP) GO TO 75
C DOES INITIAL GROUND RAY GO UPWARD ?
      IF (ANGO.LT.0.) GO TO 210
      IF ((ANGO.EQ.0.).AND.(G(LP-1).LE.0.)) GO TO 210
C IS INITIAL ANGLE ZERO ?
   75 IF(ABS(ANGO).GT.1E-3) GO TO 90
C IF INITIAL RAY IS SPLIT, ARBITRARILY MAKE DOWNARD
      IF ((G(K-1).GT.0.).AND.(G(K).LT.0.)) GO TO 80
C IF INITIAL RAY IS DOWNARD , DECREASE ANGO SLIGHTLY
```

```
      IF ((G(K-1).LE.0.).AND.(G(K).LT.0.)) GO TO 80
C IF INITIAL RAY IS UPWARD , INCREASE ANGO SLIGHTLY
      IF ((G(K-1).GT.0.).AND.(G(K).GE.0.)) GO TO 85
C MAKE SPECIAL CALCULATION IF RAY IS HORIZONTAL
      GO TO 220
   80 ANGO=ANGO-0.01
      GO TO 90
   85 ANGO=ANGO+0.01
C INITIALIZE ANGO, ETC
   90 THO=PI/180.*ANGO
      THOD=DBLE(THO)
      CSTHD=DCOS(THOD)
      CSTH=SNGL(CSTHD)
      CVD=DBLE(V(NSD))/CSTHD
      CV=SNGL(CVD)
      SITH=SIN(THO)
      SITHO=SITH
      X=0.0
      X1=0.0
      X2=0.0
      KV1=0
      KV2=0
      IBUG= 90; IF(IPDB.EQ.1) WRITE(6,888) IBUG,ANGO,SITH,CSTH,CV
C CALCULATE ONE LAYER
  100 IBUG=100; IF(IPDB.EQ.1) WRITE(6,888) IBUG,V(K),SITH,SITH2,X,X1,X2
      IF (SITH.LT.0.) GO TO 110
C       IF RAY GOES UPWARD BEYOND LIMIT
      IF(K.LE.1) GO TO 205
  105 K=K-1
      DIR=1.
      GRAD=G(K)
      GO TO 120
C DOWNARD-GOING RAY
  110 IF (K.LT.LP) GO TO 115
C REFLECTION OFF GROUND
      SITH2=-SITH2
      IF (KV1.NE.0) KV2=LP ; IF (KV1.EQ.0) KV1=LP
      GO TO 140
  115 GRAD=G(K)
      K=K+1
      DIR=-1
C DISTANCE CALCULATION; K = NEXT LAYER
C ISO-VELOCITY ?
  120 IF (GRAD.EQ.0.) GO TO 125
      VKD=DBLE(V(K))
      IBUG=120; IF(IPDB.EQ.1) WRITE(6,888) IBUG,V(K),CV
C VERTEX IN LAYER K ?
      IF (VKD.GT.CVD) GO TO 130
      IF (VKD.EQ.CVD) GO TO 205
      CSTH2=SNGL(VKD/CVD)
```

```
      SITH2=DIR*SQRT((1.-CSTH2)*(1.+CSTH2))
      DX=CV/GRAD*(SITH2-SITH)
      GO TO 135
C ISO-VELOCITY CALCULATION
  125 ID=DIR
      LAST=K+ID
      SITH2=SITH
      CSTH2=CSTH
      DX=(D(LAST)-D(K))*CSTH2/SITH2
      GO TO 135
C VERTEX CALCULATION
  130 ID=DIR
      K=K+ID
      IF (KV1.NE.0) KV2=K ; IF (KV1.EQ.0) KV1=K
      SITH2=-SITH
      CSTH2=CSTH
      DX=2.*CV/GRAD*SITH2
  135 X=X+DX/3
C CHECK RAY POSITION
C RAY AT TARGET DEPTH ?
      IF (K.NE.NTD) GO TO 140
      IF (X1.GT.0.) X2=X
      IF (X1.EQ.0.) X1=X
C RAY RETURNED TO SOURCE DEPTH ?
  140 IBUG=140; IF(IPDB.EQ.1) WRITE(6,888) IBUG,V(K),SITH,SITH2,X,X1,X2
      IF ((K.EQ.NSD).AND.(SITH0*SITH2.GT.0.)) GO TO 145
      IF((X.GT.(1.5*RYARD)).AND.(X1.EQ.0.)) GO TO 205
      SITH=SITH2
      CSTH=CSTH2
      GO TO 100
C CYCLE COMPLETED
  145 WL=X
C CHECK 1ST INTERSECTION
      IF (X1.EQ.0.) GO TO 205
      NCYC=0
      ERRA=X1-RYARD
  150 ERRB=ERRA+WL
C MINIMUM ERROR NCYC ?
      IF (ABS(ERRB).GE.ABS(ERRA)) GO TO 155
      ERRA=ERRB
      NCYC=NCYC+1
      IF (NCYC.LT.50) GO TO 150
      KIND=1
      IF (IPRINT.EQ.1) WRITE(6,902) ANG0,KV1,KV2,NCYC,KIND
      GO TO 205
C 1ST RAY ?
  155 IF (IRAY.EQ.0) GO TO 160
C THIS RAY SAME AS LAST ?
      IF ((NCYC.EQ.ICYC).AND.(KV1.EQ.IV1).AND.(KV2.EQ.IV2)) GO TO 170
C IF NEW RAY, CALCULATE INTENSITY FOR LAST RAY
```

```
      GO TO 280
  160 IRAY=IRAY+1
      ICYC=NCYC
      IV1=KV1
      IV2=KV2
      ERRIY=RYARD*1E60
      ERRIZ=ERRIY
  165 ANGI=ANGO
      ERRI=ERRA
      GO TO 175
  170 ERRIX=ERRIY
      ERRIY=ERRIZ
      ERRIZ=ABS(ERRA)
C RANGE ERROR PASS A MAX ?
      IF ((ERRIX.LT.ERRIY).AND.(ERRIZ.LT.ERRIY)) GO TO 280
C THIS RAY CLOSER TO TARGET THAN LAST ?
      IF (ABS(ERRA).LT.ABS(ERRI)) GO TO 165
C CHECK 2ND INTERSECTION
  175 IF (X2.EQ.0.) GO TO 205
      NCYC=0
      ERRA=X2-RYARD
  180 ERRB=ERRA+WL
      IF (ABS(ERRB).GE.ABS(ERRA)) GO TO 185
      ERRA=ERRB
      NCYC=NCYC+1
      IF (NCYC.LT.50) GO TO 180
      KIND=2
      IF (IPRINT.EQ.1) WRITE(5,902) ANGO,KV1,KV2,NCYC,KIND
      GO TO 205
  185 IF(JRAY.EQ.0) GO TO 190
      IF ((NCYC.EQ.JCYC).AND.(KV1.EQ.JV1).AND.(KV2.EQ.JV2)) GO TO 200
      GO TO 285
  190 JRAY=JRAY+1
      JCYC=NCYC
      JV1=KV1
      JV2=KV2
      ERRJY=RYARD*1E60
      ERRJZ=ERRJY
  195 ANGJ=ANGO
      ERRJ=ERRA
      GO TO 205
  200 ERRJX=ERRJY
      ERRJY=ERRJZ
      ERRJZ=ABS(ERRA)
      IF ((ERRJX.LT.ERRJY).AND.(ERRJZ.LT.ERRJY)) GO TO 285
      IF (ABS(ERRA).LT.ABS(ERRJ)) GO TO 195
C DECREMENT ANGO
  205 NSTEP=NSTEP+1
      STEPN=FLOAT(NSTEP)
      ANGO=ANGMAX-STEPN*STEP
```

```
C DECREMENTED THRU THE RANGE ?
      IF (ANGO.GE.ANGMIN) GO TO 65
C CONVERGE LAST I AND J RAYS
  210 IJ=1
C     IF NO RAYS FOUND
      IF((IRAY+JRAY+NRAY).EQ.0) RETURN
      GO TO 280
C HORIZONTAL RAY CALCULATION
  220 IF (NSD.NE.NTD) GO TO 205
      ERR=0.
      S=RANGE
      TIM=3.*RYARD/V(K)
      SPL=20.*ALOG10(RYARD)
      ATTN=ALPHA*S
      NS=0
      NB=0
      LCYC=0
      KIND=0
      LV1=K
      LV2=K
      WRITE(6,951) ANGO,ERR,NS,NB,S,TIM,SPL,ATTN,LV1,LV2,LCYC,KIND
      GO TO 205
C ZERO IN ON TARGET AND CALCULATE INTENSITY LOSS
  280 KIND=1
      ANGL=ANGI
      LCYC=ICYC
      ERRL=ERRI
      LV1=IV1
      LV2=IV2
      GO TO 290
  285 KIND=2
      ANGL=ANGJ
      LCYC=JCYC
      ERRL=ERRJ
      LV1=JV1
      LV2=JV2
      IF (IJ.EQ.1) IJ=2
  290 THOD=PID*DBLE(ANGL)/180D0
      THO=SNGL(THOD)
      ERRP=2.*ERRL
      IVTX=0
  295 NS=0
      NB=0
      INT=0
      MV1=0
      MV2=0
      ANGB=0.0
      ANGS=0.0
      X=0.0
      XT=0.0
```

```
      S=0.0
      TIM=0.0
      SUM=0.0
      XD=0D0
      XTD=0D0
      SUMD=0D0
      SITHD=DSIN(THOD)
      CSTHD=DCOS(THOD)
      CVD=DBLE(V(NSD))/CSTHD
      CV=SNGL(CVD)
      SITHO=SNGL(SITHD)
      CSTHO=SNGL(CSTHD)
      TNTHO=SITHO/CSTHO
      ANGLO=180./PI*THO
      TH=THO
      SITH=SITHO
      CSTH=CSTHO
      K=NSD
  300 IF (SITH.LT.0.) GO TO 310
      IF (K.LE.1) GO TO 205
  305 K=K-1
      DIR=1.
      GRAD=G(K)
      GO TO 320
  310 IF (K.LT.LP) GO TO 315
      NB=1
      IF (MV1.NE.0) MV2=LP ; IF (MV1.EQ.0) MV1=LP
      S2=SITH*SITH
      ANGB=180./PI*ATAN(SQRT(S2/(1.-S2)))
      SITH2D=-SITH2D
      SITH2=-SITH2
      TH2=-TH2
      GO TO 355
  315 GRAD=G(K)
      K=K+1
      DIR=-1.
  320 IF (GRAD.EQ.0.) GO TO 335
      VKD=DBLE(V(K))
      IF (VKD.GT.CVD) GO TO 340
      IF (VKD.EQ.CVD) GO TO 379
      CSTH2D=VKD/CVD
      SITH2D=DBLE(DIR)/CVD*DSQRT((CVD-VKD)*(CVD+VKD))
      CSTH2=SNGL(CSTH2D)
      SITH2=SNGL(SITH2D)
      TH2=ATAN(SITH2/CSTH2)
  325 DXD=CVD*(SITH2D-SITHD)/DBLE(GRAD)
      DX=SNGL(DXD)
      DS=CV/GRAD*(TH2-TH)
      ARG=SNGL((1D0+SITH2D)/(1D0-SITH2D)*(1D0-SITHD)/(1D0+SITHD))
      DTIM=0.5/GRAD*ALOG(ARG)
```

```
330 DSUMD=DXD/SITH2D/SITHD
    XD=XD+DXD/3D0
    X=SNGL(XD)
    S=S+DS/3.
    TIM=TIM+DTIM
    SUMD=SUMD+DSUMD/3D0
    SUM=SNGL(SUMD)
    GO TO 345
335 ID=DIR
    LAST=K+ID
    TH2=TH
    SITH2=SITH
    CSTH2=CSTH
    SITH2D=SITHD
    CSTH2D=CSTHD
    H=D(LAST)-D(K)
    DXD=DBLE(H)*CSTH2D/SITH2D
    DX=SNGL(DXD)
    DS=SQRT(DX*DX+H*H)
    DTIM=DS/V(K)
    GO TO 330
340 ID=DIR
    K=K+ID
    TH2=-TH
    SITH2=-SITH
    CSTH2=CSTH
    SITH2D=-SITHD
    CSTH2D=CSTHD
    IF (MV1.NE.0) MV2=K ; IF (MV1.EQ.0) MV1=K
    GO TO 325
345 IF (K.NE.NTD) GO TO 355
    INT=INT+1
    IF (INT.NE.KIND) GO TO 355
    XTD=XD
    XT=SNGL(XTD)
    ST=S
    TIMT=TIM
    SUMT=SUM
    NST=NS
    NBT=NB
355 IF ((K.EQ.NSD).AND.(THO*TH2.GT.0.)) GO TO 360
    IF  (((X.GT.(1.5*RYARD)).AND.(INT.EQ.0)).OR.(INT.GT.2)) GO TO 375
    TH=TH2
    SITH=SITH2
    CSTH=CSTH2
    SITHD=SITH2D
    CSTHD=CSTH2D
    GO TO 300
360 CYCL=FLOAT(LCYC)
    XD=XTD+XD*DBLE(CYCL)
```

```
      X=SNGL(XD)
      ERR=SNGL(XD-RYARDD)
      IF ((MV1.NE.LV1).OR.(MV2.NE.LV2)) GO TO 378
      IF (ABS(ERR).GE.ABS(ERRP)) GO TO 370
      SUM=SUMT+CYCL*SUM
      IF (ABS(ERR).LE.ERRMX) GO TO 365
      DXDTH=-SUM*TNTHO
      DTHO=ERR/DXDTH
      DANGO=180./PI*DTHO
      IF (ABS(DANGO).GT.(10.*STEP)) GO TO 377
      THOD=THOD-DBLE(DTHO)
      THO=SNGL(THOD)
      ERRP=ERR
      GO TO 295
  365 S=ST+CYCL*S
      TIM=TIM+CYCL*TIM
      SPL=10.*ALOG10(ABS(X*SITH2*TNTHO*SUM/CSTHO))
      NS=NST+LCYC*NS
      NB=NBT+LCYC*NB
      S=1E-3*S
      ATTN=ALPHA*S
      IF (IPRINT.EQ.1) WRITE(6,951)
     1 ANGLO,ERR,NS,NB,S,TIM,SPL,ATTN,LV1,LV2,LCYC,KIND
      NRAY=NRAY+1
      TT(NRAY)=TIM
      DB(NRAY)=SPL
      ATN(NRAY)=ATTN
      ANO(NRAY)=ANGLO
      ANS(NRAY)=ANGS
      ANB(NRAY)=ANGB
      LS(NRAY)=NS
      LB(NRAY)=NB
      IF (NRAY.LT.NRAYMX) GO TO 380
      WRITE(6,805)
      RETURN
  370 IF (IPRINT.EQ.1) WRITE(6,952) ANGL,ANGLO,LV1,LV2,LCYC,KIND
      GO TO 380
  375 IF (IPRINT.EQ.1) WRITE(6,953) ANGL,ANGLO,LV1,LV2,LCYC,KIND
      GO TO 380
  377 IF ( IPRINT.EQ.1) WRITE(6,954) ANGL,DANGO,LV1,LV2,LCYC,KIND
      GO TO 380
  378 IF (IVTX.GE.3) GO TO 379
      IVTX=IVTX+1
      DTHO=DTHO/2.
      THOD=THOD+DBLE(DTHO)
      THO=SNGL(THOD)
      GO TO 295
  379 IF (IPRINT.EQ.1) WRITE(6,955) ANGL,ANGLO,LV1,LV2,LCYC,KIND
  380 IF ((IJ.EQ.1).AND.(JRAY.GT.0)) GO TO 285
      IF ((IJ.EQ.2).OR.((IJ.EQ.1).AND.(JRAY.EQ.0))) RETURN
```

```
      IF (KIND.EQ.1) GO TO 160
      GO TO 190
  800 FORMAT(1H ,I2,2F10.2,F12.3)
  801 FORMAT(1H ,32HSVP WITH SOURCE AND TARGET ADDED//)
  802 FORMAT(1H1,12HSOURCE DEPTH,F8.1,3H FT/
     1       1H ,12HTARGET DEPTH,F8.1,3H FT/
     2       1H ,5HRANGE,F8.3,5H KYDS//
     3       1H ,9HMAX ANGLE,F6.1,4H DEG/
     4       1H ,9HMIN ANGLE,F6.1,4H DEG//
     5       1H ,9HFREQUENCY,F7.3,4H KHZ/
     6       1H ,10HATTN COEFF,1PE10.2,7H DB/KYD////)
  805 FORMAT(1H0,48H*** FOUND TOO MANY RAYS - DECREASE ANGMAX,ANGMIN)
  902 FORMAT(1H ,F7.3,21H CYCLE LIMIT EXCEEDED,32X,I5,I7,I7,I6//)
  950 FORMAT(1H1,17X,37HTABLE OF SOUND RAY PATH INTERSECTIONS//
     1       1H ,37HINITIAL RANGE    NUMBER OF RAY          ,
     2           44HTRAVEL SPREADING   ATTN  1ST    2ND   NUMBER/
     3       1H ,38HANGLE  ERROR   REFLECTIONS  LENGTH        ,
     4           47HTIME    LOSS      LOSS VERTEX VERTEX   OF   RAY/
     5       1H ,37H (DEG)  (YDS) SURFACE BOTTOM (KYDS)       ,
     6           49H(SECS)  (DB)      (DB) LAYER  LAYER  CYCLES TYPE//)
  951 FORMAT(1H ,F7.3,F6.1,I6,I7,F9.2,F8.3,F8.2,F9.3,I5,I7,I7,I6//)
  952 FORMAT(1H ,F7.3,16H RAY DIVERGED AT,F9.3,4H DEG,24X,I5,I7,I7,I6//)
  953 FORMAT(1H ,F7.3,12H RAY LOST AT,F9.3,4H DEG,28X,I5,I7,I7,I6//)
  954 FORMAT(1H ,F7.3,15H ATTEMPTED JUMP,F9.3,4H DEG,25X,I5,I7,I7,I6//)
  955 FORMAT(1H ,F7.3,15H DIFF VERTEX AT,F9.3,4H DEG,25X,I5,I7,I7,I6//)
  888 FORMAT(1H0,I8/10(1PE13.5))
      END
      FUNCTION BLOS(F,P,THETA)
C CALCULATES BOTTOM LOSS FROM NUWC TECH NOTE 10 (DEC 67).
C     F = FREQ (KHZ), P = POROSITY, THRTA = BOTTOM GRAZING ANGLE
      DIMENSION ABTLOS(14)
      DATA ABTLOS(1),ABTLOS(2),ABTLOS(3),ABTLOS(4),ABTLOS(5),
     1 ABTLOS(6),ABTLOS(7),ABTLOS(8),ABTLOS(9),ABTLOS(10),ABTLOS(11),
     2 ABTLOS(12),ABTLOS(13),ABTLOS(14) /.16,.67,1.,1.18,1.31,1.43,1.52,
     3 1.61,1.7,1.76,1.82,1.88,1.94,2./
      BLOS=0.0
      IF(P.LT.0.01) RETURN
      IF(F.GT.0.1) GO TO 15
      FUNU=0.16
      GO TO 40
   15 IF(F.LT.6.5) GO TO 20
      FUNU=2.0
      GO TO 40
   20 DO 30 I=1,7
      XI=I
      IF(XI*0.5.GT.F) GO TO 35
   30 CONTINUE
   35 IF(I.EQ.1) GO TO 45
      FUNU=ABTLOS(I)+(ABTLOS(I+1)-ABTLOS(I))*(F-(XI-1.)*0.5)/0.5
      GO TO 40
```

```
   45 FUNU=ABTLOS(1)+(ABTLOS(2)-ABTLOS(1))*(F-0.1)/0.4
   40 ARG=1.5/P*ALOG(P*THETA/13.74)
      ARG=EXP(ARG)
      BLOS=(3.7+17.5*(P-.27))*FUNU*(TANH(ARG)+(1.0-P/0.27)/12.5*
     1 (THETA/90.0)**2)
      RETURN
      END
      SUBROUTINE INTOUT(NRAY,TWIN,XIOM)
C SUMS INTENSITY IN MOVING WINDOW TWIN SECONDS LONG.
      COMMON /R/TT(99),DB(99),ATN(99),ANGO(99),ANGS(99),ANGB(99)
      COMMON /R/NS(99),NB(99)
      COMMON /P/TLOSS(99)
      DIMENSION XINT(99)
      XLN10=0.23025851
      XIOM=-400.
      WRITE(6,400) TWIN
      DO 10 I=1,NRAY
   10 XINT(I)=EXP(-XLN10*TLOSS(I))
      SUM=0.
      K1=1
      K2=1
   15 IF(TT(K1)-TT(K2)+TWIN) 30,30,20
   20 T2=TT(K2)
      SUM=SUM+XINT(K2)
      L2=K2
      K2=K2+1
      GO TO 40
   30 T2=TT(K1)+TWIN
      SUM=SUM-XINT(K1)
      L1=K1
      K1=K1+1
      IF((TT(L1)-TT(K2)+TWIN).EQ.0.) GO TO 20
   40 RCV=10.*ALOG10(ABS(SUM+1E-30))
      WRITE(6,401) T2,K1,L2,RCV
      XIOM=AMAX1(XIOM,RCV)
      IF(K2.LE.NRAY) GO TO 15
      WRITE(6,450) XIOM
  450 FORMAT(1H ,24HMAX INTEGRATOR OUTPUT = ,F10.2,/////)
      RETURN
  400 FORMAT(///,2X,17HINTEGRATOR OUTPUT///
     1 1H ,11HTIME WINDOW,F6.3,4H SEC//
     2 1H ,23H TIME   1ST 2ND  OUTPUT/
     3 1H ,22H (SEC)  RAY RAY   (DB)//)
  401 FORMAT(1H ,F6.4,I4,I4,F9.2)
      END
      SUBROUTINE TTORD(NRAY)
C ORDERS EIGENRAYS BY TRAVEL TIME
      COMMON /R/TT(99),DB(99),ATN(99),ANGO(99),ANGS(99),ANGB(99)
      COMMON /R/NS(99),NB(99)
      IE=NRAY-1
```

```
      DO 25 I=1,IE
      JS=I+1
      DO 25 J=JS,NRAY
      IF (TT(J).GE.TT(I)) GO TO 25
      TEMP 1=TT(I)
      TEMP 2=DB(I)
      TEMP 3=ATN(I)
      TEMP 4=ANGO(I)
      TEMP 5=ANGS(I)
      TEMP 6=ANGB(I)
      NTEMP 1=NS(I)
      NTEMP 2=NB(I)
      TT(I)=TT(J)
      DB(I)=DB(J)
      ATN(I)=ATN(J)
      ANGO(I)=ANGO(J)
      ANGS(I)=ANGS(J)
      ANGB(I)=ANGB(J)
      NS(I)=NS(J)
      NB(I)=NB(J)
      TT(J)=TEMP1
      DB(J)=TEMP2
      ATN(J)=TEMP3
      ANGO(J)=TEMP4
      ANGS(J)=TEMP5
      ANGB(J)=TEMP6
      NS(J)=NTEMP1
      NB(J)=NTEMP2
   25 CONTINUE
      RETURN
      END
      SUBROUTINE SSP(NP,NDFT,NVFT,NTF,NWV,IVEL,ITEMP,IWV)
C CALCULATE SOUND SPEED PROFILE FROM BERANAK
C     D=DEPTH
C     G=SOUND SPEED GRADIENT
C     V=SOUND SPEED
C     T=TEMP
C     WV=WIND VELOCITY
      COMMON/SX/D(100),V(100),G(99),T(100),WV(100)
C SOUND SPEED GIVEN ?
      IF (IVEL.EQ.1) GO TO 50
C TEMP IN DEG F ?
      IF (NTF.NE.1) GO TO 10
C CONVERT TEMP TO DEG C
      DO 5 I=1,NP
    5 T(I)=5./9.*(T(I)-32.)
C DEPTH IN FT ?
   10 IF (NDFT.NE.1) GO TO 20
C CONVERT DEPTH TO METERS
      DO 15 I=1,NP
```

```
   15 D(I)=D(I)*0.3048
C WIND VELOCITY GIVEN?
   20 IF(IWV.EQ.1) GO TO 27
C ASSUME NO WIND VELOCITY
      DO 25 I=1,NP
   25 WV(I)=0.0
C     WIND VELOCITY IN METERS/SEC?
 27   IF(NWV.EQ.0) GO TO 30
C     CONVERT TO METERS/SEC
      DO 26 I=1,NP
 26   WV(I)=WV(I)*.304878
C CALCULATE SOUND SPEED
   30 DO 35 I=1,NP
 35   V(I)=331.4*(SQRT(1.0+0.00366*T(I)))+WV(I)
C CALCULATE SOUND SPEED GRADIENT
      DO 40 I=2,NP
      J=I-1
   40 G(J)=(V(I)-V(J))/(D(I)-D(J))
C IF VELOCITY INPUT IN MKS OUTPUT IN MKS
      IF(NVFT.EQ.0) GO TO 45
 46   DO 47 I=1,NP
      V(I)=V(I)*3.28084
      D(I)=D(I)*3.28084
      WV(I)=WV(I)*3.28084
 47   T(I)=9./5.*T(I)+32.
      IF(NVFT.EQ.0) RETURN
C PRINT PROFILE INCLUDING TEMP AND WV
      WRITE(6,801)
      IF=NP-1
      WRITE(6,802) (I,D(I),V(I),T(I),WV(I),G(I),I=1,IF)
      WRITE(6,802) NP,D(NP),V(NP),T(NP),WV(NP)
      RETURN
C     MKS OUTPUT
 45   WRITE(6,805)
      IF=NP-1
      WRITE(6,802) (I,D(I),V(I),T(I),WV(I),G(I),I=1,IF)
      WRITE(6,802) NP,D(NP),V(NP),T(NP),WV(NP)
      GO TO 46
C SOUND SPEED IN FPS ?
   50 IF (NVFT.EQ.1) GO TO 60
C     OUTPUT IN MKS UNITS
      IF(NDFT) 53,70,53
 53   DO 51 I=1,NP
 51   D(I)=D(I)*.304878
      NDFT=0
      GO TO 70
C CONVERT SPEED TO FPS
 52   DO 55 I=1,NP
   55 V(I)=V(I)*3.28084
C DEPTH IN FT ?
```

```
   60 IF (NDFT.EQ.1) GO TO 70
C CONVERT TO FEET
      DO 65 I=1,NP
   65 D(I)=D(I)*3.28084
      IF(NVFT.EQ.0)RETURN
C CALCULATE SOUND SPEED GRADIENT
   70 DO 75 I=2,NP
      J=I-1
   75 G(J)=(V(I)-V(J))/(D(I)-D(J))
C PRINT PROFILE
C       MKS OUTPUT OR BES
      IF(NVFT.EQ.1) GO TO 80
      WRITE(6,806)
      IE=NP-1
      WRITE(6,804) (I,D(I),V(I),G(I),I=1,IE)
      WRITE(6,804) NP,D(NP),V(NP)
      GO TO 52
 80   WRITE(6,803)
      IE=NP-1
      WRITE(6,804) (I,D(I),V(I),G(I),I=1,IE)
      WRITE(6,804) NP,D(NP),V(NP)
      RETURN
  801 FORMAT(20X,19HSOUND SPEED PROFILE//
     1       8X,41HDEPTH    SPEED    GRADIENT    TEMP      WV/
     2       9X,44H(FT)  (FT/SEC)   (FPS/FT)  (DEG F)  (FT/SEC)//)
  802 FORMAT(1H ,I2,1PE10.3,0PF10.3,11X,2F8.3/21X,1PE11.3)
  803 FORMAT(10X,19HSOUND SPEED PROFILE//
     1       8X,26HDEPTH    SPEED    GRADIENT/
     2       9X,25H(FT)  (FT/SEC)   (FPS/FT)//)
  804 FORMAT(1H ,I2,0PE10.3,F10.3/21X,1PE11.3)
 805  FORMAT(20X,19HSOUND SPEED PROFILE//
     1       8X,41HDEPTH    SPEED    GRADIENT    TEMP      WV/
     2       9X,42H(M)    (M/SEC)   (M/S/M)   (DEG C)  (M/SEC)//)
 806  FORMAT(10X,19HSOUND SPEED PROFILE//
     1       8X,26HDEPTH    SPEED    GRADIENT/
     2       9X,25H(M)    (M/SEC)    (M/S/M) //)
      END
```

Appendix B

```
C                                   SRP-PLOT 2 PROGRAM
C                                        (4/6/73)
C    COMPUTES SOUND RAY PATHS AND WRITES TAPE OR PUNCHES CARDS FOR
C    PLOTTING PATHS USING CALCOMP.
C    WILL TAKE UP TO 150 BOTTOM DEPTH COORDINATES AND 5 SVPS WITH A
C    MAXIMUM OF 200 POINTS IN A SVP. EACH SVP MUST REACH THE SAME
C    MAXIMUM DEPTH AS THE ADJACENT GIVEN SVPS.
      DOUBLE.PRECISION ALPH,ANGLE(3000),ANGX,AQ,B,BDEP(150),BETA,BQ,
     1 BRAN(150),CQ,CSTH,CSTHX,CSTH2,D(6,100),DD,DDEPTH,DELR,DELX,DELY,
     2 DEP(3000),DEPMIN,DISC,DM,DR,DRFT,G(6,100),GAM,GC,GI,GI2,GRAD,P,
     3 P2,P3,P4,R(3000),RA,RC,RC2,RMAX,RMAXL,RSVP(6),SAL(6,100),SC,SD,
     4 SITH,SITHX,SITH2,SLOPE(150),SR,T(6,100),TATH,TANTH,TANTHX,TC,TC2,
     5 TC3,TC4,TD,TEMP,TEM2,TEM3,TEM4,TF(6,100),THONE,TR,V(6,100),VC,VP,
     6 VS,VSTP,VT,X,XBRAN,XP,Y,WV(6,100)
      INTEGER*2 MA(1000),MB(1000),MC(1000)
      INTEGER IGSVP(6,250),NPSVP(6),GRAPH/0/
      REAL DQ(6,1000),QD(6),PI/3.141593/
      LOGICAL*1 TITLE(40),ALPD(20),TSVP(5)
      FUNC(A,B,C,D,E) = A + (B - C)*(E - A)/(D - C)
      DATA TSVP/'S','V','P',' '/
      CALL NOPRQ
      CALL INITQ(MA,MB,MC,DQ,1000)
   10 READ(5,701) TITLE
      HTMIN=0.00
      HTMAX=100.0
C
C
      READ(5,702) NRSVPS,NRBOT,METER,NAUT,DELR,NOUT,NEWSVP,NOPR,NOAD,
     1 NRAN,RANINC,RANL,NDEP,DEPINC,DEPL,NSV,SVINC,SVL,SVMIN,RANGE
      IF(NOUT.LT.71) GO TO 14
      CALL STSWQ(564,71)
   14 IF(NOAD.LT.1) GO TO 16
      READ(5,701) ALPD
   16 IF(NEWSVP.LT.1) GO TO 133
      DO 128 NR = 1,NRSVPS
      READ(5,703) NODEP,NODFT,NOTEMP,NOTF,NOVEL,NOVFT,NOSAL
      NSVP = 1
   20 READ(5,704) D(NR,NSVP),TF(NR,NSVP),WV(NR,NSVP),SAL(NR,NSVP),
     1 IGSVP(NR,NSVP),NOMO
      D(NR,NSVP)=HTMAX-(D(NR,NSVP)-HTMIN)
      IF(NOMO.GT.0) GO TO 24
      NSVP = NSVP + 1
      GO TO 20
 24   DO 97 I=1,NSVP
  97  V(NR,I)=331.4*(DSQRT(1.0+0.00366*TF(NR,I)))+WV(NR,I)
   98 J = NSVP - 1
C
C   CALCULATE VELOCITY GRADIENTS
C
      DO 103 I = 1,J
```

```
      G(NR,I) = (V(NR,I+1) - V(NR,I))/(D(NR,I+1) - D(NR,I))
      IF(DABS(G(NR,I)).GE.0.0002) GO TO 103
      G(NR,I) = 0.0
  103 CONTINUE
      G(NR,NSVP) = G(NR,NSVP-1)
      IB = 1
      NPAGE = 1
  107 WRITE(6,705) NPAGE,TITLE,NR,NSVP
      IF(NOVEL.LT.1) GO TO 110
      WRITE(6,706)
  110 WRITE(6,707) NR
      IL = IB + 21
      IF(IL.LE.J) GO TO 114
      IL = J
  114 IF(NOTEMP.GT.0) GO TO 117
      WRITE(6,708) (D(NR,I),TF(NR,I),SAL(NR,I),V(NR,I),G(NR,I),
     1 I = IB,IL)
      GO TO 118
  117 WRITE(6,709) (D(NR,I),V(NR,I),G(NR,I), I = IB,IL)
  118 IB = IL + 1
      IF(IB.GT.J) GO TO 123
      WRITE(6,710)
      NPAGE = NPAGE + 1
      GO TO 107
  123 IF(NOTEMP.GT.0) GO TO 126
      WRITE(6,708) D(NR,NSVP),TF(NR,NSVP),SAL(NR,NSVP),V(NR,NSVP)
      GO TO 127
  126 WRITE(6,709) D(NR,NSVP),V(NR,NSVP)
  127 NPSVP(NR) = NSVP
  128 CONTINUE
C
C  LABEL PLOTS          *********************************
C
      DO 132 I = 1,NSVP
      D(NRSVPS+1,I) = D(NRSVPS,I)
      G(NRSVPS+1,I) = G(NRSVPS,I)
  132 V(NRSVPS+1,I) = V(NRSVPS,I)
  133 DISH = 0.5*(RANL + FLOAT(NRSVPS)*(SVL + 1.0))
      CALL XFSTQ(DISH,1.6,0.24,0.26,0.0,0.0,QD)
      CALL LABLQ(TITLE,-40,QD,GRAPH,40)
      DISSR = RANL/2.0
      IF(NOAD.LT.1) GO TO 143
      CALL XFSTQ(DISSR - 1.7,0.95,0.18,0.20,0.0,0.0,QD)
      CALL LABLQ('SOUND RAY PATHS - INTENSITY CONTOURS',-38,QD,GRAPH,38)
      CALL XFSTQ(DISSR + 3.7,0.95,0.15,0.16,0.0,0.0,QD)
      CALL LABLQ(ALPD,-20,QD,GRAPH,20)
      GO TO 145
  143 CALL XFSTQ(DISSR,0.95,0.18,0.20,0.0,0.0,QD)
      CALL LABLQ('SOUND RAY PATHS',-15,QD,GRAPH,15)
  145 CALL XFSTQ(DISSR,0.6,0.18,0.20,0.0,0.0,QD)
```

```
      IF(NAUT.EQ.0) GO TO 149
      CALL LABLQ('RANGE - NAUTICAL MILES',-22,QD,GRAPH,22)
      GO TO 150
  149 CALL LABLQ('RANGE - METERS',-14,QD,GRAPH,14)
  150 CALL XFSTQ(0.0,0.0,1.0,1.0,0.0,0.0,QD)
      CALL AXISQ(RANL,NRAN,0.0,RANINC,-0.13,-1,QD,GRAPH)
      CALL DISPQ(GRAPH,200.0)
      CALL REMVQ(GRAPH)
      DELSV = FLOAT(NSV)*SVINC/SVL
      DELDEP = FLOAT(NDEP)*DEPINC/DEPL
      DELRAN = FLOAT(NRAN)*RANINC/RANL
C  DRAW SOUND VELOCITY PROFILES
      DO 194 M = 1,NRSVPS
      AM = M - 1
      DISSV = RANL + SVL/2.0 + 1.0 + AM*(SVL + 1.0)
      CALL XFSTQ(DISSV,0.99,0.18,0.20,0.0,0.0,QD)
      IF(NRSVPS.LT.2) GO TO 165
      CALL EDINCH(M,TSVP(5),1)
      CALL LABLQ(TSVP,-5,QD,GRAPH,5)
      GO TO 166
  165 CALL LABLQ('SVP',-3,QD,GRAPH,3)
  166 CALL XFSTQ(DISSV,0.55,0.18,0.20,0.0,0.0,QD)
      CALL LABLQ('VELOCITY -  M/SEC',-17,QD,GRAPH,17)
      DISVP = RANL + 1.0 + AM*(SVL + 1.0)
      CALL XFSTQ(DISVP,0.0,1.0,1.0,0.0,0.0,QD)
      CALL AXISQ(SVL,NSV,SVMIN,SVINC,-0.13,-1,QD,GRAPH)
      DISSD = RANL + SVL + 1.0 + AM*(SVL + 1.0)
      CALL XFSTQ(DISSD,0.0,1.0,1.0,1.5*PI,0.0,QD)
      CALL AXISQ(DEPL,NDEP,0.0,DEPINC,-0.13,-1,QD,GRAPH)
      DISDH = DISSD + 0.52
      DISD = DEPL/2.0
      CALL XFSTQ(DISDH,-DISD,0.18,0.20,1.5*PI,0.0,QD)
      CALL LABLQ('HEIGHT - METERS',-15,QD,GRAPH,15)
      DSVP = RANL + (V(M,1) - SVMIN)/DELSV + 1.0 + AM*(SVL+1.0)
      CALL XFSTQ(DSVP,-0.02,0.18,0.20,0.0,0.0,QD)
      CALL LABLQ('0',-1,QD,GRAPH,1)
      CALL ADPTQ(DSVP,0.0,1,GRAPH)
      NSVP = NPSVP(M)
      DO 191 I = 2,NSVP
      DSVP = RANL + (V(M,I) - SVMIN)/DELSV + 1.0 + AM*(SVL + 1.0)
      DEPP = - D(M,I)/DELDEP
      CALL ADPTQ(DSVP,DEPP,1,GRAPH)
      IF(IGSVP(M,I).LT.1) GO TO 191
      CALL XFSTQ(DSVP,DEPP - 0.02,0.18,0.20,0.0,0.0,QD)
      CALL LABLQ('0',-1,QD,GRAPH,1)
      CALL ADPTQ(DSVP,DEPP,1,GRAPH)
  191 CONTINUE
      CALL DISPQ(GRAPH,200.0)
      CALL REMVQ(GRAPH)
  194 CONTINUE
```

```
      CALL ADPTQ(RANL,0.0,0,GRAPH)
      IF(RANGE.LE.0.0) GO TO 199
      CALL ADPTQ(RANGE/DELRAN,-DEPL,0,GRAPH)
      CALL ADPTQ(RANGE/DELRAN,0.0,1,GRAPH)
  199 CALL XFSTQ(0.0,0.0,1.0,1.0,1.5*PI,0.0,QD)
      CALL AXISQ(DEPL,NDEP,0.0,DEPINC,0.13,-1,QD,GRAPH)
      CALL XFSTQ(-0.66,-DISD,0.18,0.20,1.5*PI,0.0,QD)
      CALL LABLQ('HEIGHT - METERS',-15,QD,GRAPH,15)
      IF(NRSVPS.LT.2) GO TO 232
      READ(5,711) (RSVP(I), I = 1,NRSVPS)
      DO 209 I = 2,NRSVPS
      RSV = RSVP(I)/DELRAN
      CALL ADPTQ(RSV,- DEPL - 1.0,0,GRAPH)
      CALL ADPTQ(RSV,0.0,1,GRAPH)
  209 CONTINUE
      DISVP = DEPL + 1.0
      NRSVPM = NRSVPS - 1
      DO 217 I = 1,NRSVPM
      RSV = 0.5*(RSVP(I+1) + RSVP(I))/DELRAN
      CALL EDINCH(I,TSVP(5),1)
      CALL XFSTQ(RSV,-DISVP,0.18,0.20,0.0,0.0,QD)
      CALL LABLQ(TSVP,-5,QD,GRAPH,5)
  217 CONTINUE
      CALL EDINCH(NRSVPS,TSVP(5),1)
      RSV = 0.5*(RANL*DELRAN + RSVP(NRSVPS))/DELRAN
      CALL XFSTQ(RSV,-DISVP,0.16,0.16,0.0,0.0,QD)
      CALL LABLQ(TSVP,-5,QD,GRAPH,5)
      CALL DISPQ(GRAPH,200.0)
      CALL REMVQ(GRAPH)
      IF(NAUT.EQ.0) GO TO 227
      WRITE(6,712) TITLE
      GO TO 228
  227 WRITE(6,713) TITLE
  228 WRITE(6,714) (I,RSVP(I), I = 1,NRSVPS)
      IF(NAUT.EQ.0) GO TO 232
      DO 231 I = 1,NRSVPS
  231 RSVP(I) = 6080.0*RSVP(I)/3.0
  232 IF(NRBOT.LE.0) GO TO 264
      READ(5,715) (BDEP(I),BRAN(I), I = 1,NRBOT)
      DO 911 I=1,NRBOT
 911  BDEP(I)=HTMAX-(BDEP(I)-HTMIN)
      IF(METER.EQ.1) GO TO 238
C  LINE ADDED    BDEP STAYS AS METERS
      DO 237 I = 1,NRBOT
      ID = 328.084*BDEP(I)
  237 BDEP(I) = DFLOAT(ID)/100.0
  238 XD = 200.0*BRAN(1)/DELRAN
      YD = - 200.0*BDEP(1)/DELDEP
      CALL DRAWQ(XD,YD,0)
      DO 245 I = 2,NRBOT
```

```
      XD = 200.0*BRAN(I)/DELRAN
      YD = - 200.0*BDEP(I)/DELDEP
      CALL DRAWQ(XD,YD,1)
  245 CONTINUE
      CALL GDMPQ
      IF(NAUT.EQ.0) GO TO 250
      DO 249 I = 1,NRBOT
  249 BRAN(I) = 6080.0*BRAN(I)/3.0
  250 WRITE(6,716) TITLE
      DEPMIN = BDEP(1)
      IM = NRBOT - 1
      DO 260 I = 1,IM
      IP = I + 1
      SLOPE(I) = (BDEP(I) - BDEP(IP))/(BRAN(IP) - BRAN(I))
      XBRAN = 3.0*BRAN(I)/6080.0
      WRITE(6,717) BDEP(I),XBRAN,BRAN(I),SLOPE(I)
      IF(DEPMIN.LT.BDEP(IP)) GO TO 260
      DEPMIN = BDEP(IP)
  260 CONTINUE
      XBRAN = 3.0*BRAN(NRBOT)/6080.0
      WRITE(6,717) BDEP(NRBOT),XBRAN,BRAN(NRBOT)
      GO TO 265
  264 DEPMIN = 100000.0
  265 IF(NAUT.EQ.0) GO TO 267
      DELR = 6080.0*DELR/3.0
  267 NRSVPS = NRSVPS + 1
      NPSVP(NRSVPS) = NPSVP(NRSVPS-1)
      NRAY = 1
  270 READ(5,718) SD,NOSUR,NOBOT,RA,RMAX
      SD=HTMAX-(SD-HTMIN)
      IF(RMAX.LE.0.0) GO TO 611
      IF(NAUT.EQ.0) GO TO 274
      RMAX = 6080.0*RMAX/3.0
  274 RSVP(NRSVPS) = RMAX + 3000.0
      IF(METER.EQ.0) GO TO 278
      ID = 328.084*SD
C     SD = DFLOAT(ID)/100.0
  278 NSVP = NPSVP(1)
      DO 281 I = 1,NSVP
      IF(SD - D(1,I))286,284,281
  281 CONTINUE
      WRITE(6,719) SD,D(1,NSVP)
      GO TO 270
  284 K = I
      GO TO 311
  286 K = I
      J = I - 1
      NR = 1
  289 NPSVP(NR) = NPSVP(NR) + 1
      NSVP = NPSVP(NR)
```

```
      GC = G(NR,J)
      X = (SD - D(NR,J))/(D(NR,I) - D(NR,J))
      VC = V(NR,J) + X*(V(NR,I) - V(NR,J))
      KB = I + 1
      KOUNT = 0
      DO 303 M = KB,NSVP
      MK = NSVP - KOUNT
      MN = MK - 1
      D(NR,MK) = D(NR,MN)
      G(NR,MK) = G(NR,MN)
      V(NR,MK) = V(NR,MN)
      IGSVP(NR,MK) = IGSVP(NR,MN)
  303 KOUNT = KOUNT + 1
      D(NR,I) = SD
      G(NR,I) = GC
      V(NR,I) = VC
      IGSVP(NR,I) = 0
      NR = NR + 1
      IF(NR.GT.NRSVPS) GO TO 311
      IF(NPSVP(NR).GE.K) GO TO 289
  311 KREST = 0
      NOANG = 0
      NOFIN = 0
      NOM = 0
      NR = 1
      ANGLE(1) = RA
      DEP(1) = SD
      R(1) = 0.0
      L = 2
      NSVP = NPSVP(NR)
      THONE = 0.01745329252*RA
      CSTH = DCOS(THONE)
      SITH = DSIN(THONE)
      TANTH = SITH/CSTH
      VS = V(1,K)
      SR = CSTH/VS
      GI = G(1,K-1)
      GI2 = G(1,K)
      GO TO 335
  330 IF(K.EQ.1) GO TO 333
      GI = FUNC(G(NR,K-1),R(L-1),RSVP(NR),RSVP(NR+1),G(NR+1,K-1))
      IF(DABS(GI).LT.0.0002) GI = 0.0
  333 GI2 = FUNC(G(NR,K),R(L-1),RSVP(NR),RSVP(NR+1),G(NR+1,K))
      IF(DABS(GI2).LT.0.0002) GI2 = 0.0
  335 IF(SITH)348,336,338
  336 IF     T.NPSVP(NR).AND.GI2.LT.0.0) GO TO 348
     .       .E.1.OR.GI.LE.0.0) GO TO 366
  338 C    + 1.0
      K = K - 1
      IF(K.GT.0) GO TO 346
```

```
      IF(NOSUR.GT.0) GO TO 526
      K = K + 1
  343 SITH = - SITH
      TANTH = - TANTH
      GO TO 330
  346 GRAD = GI
      GO TO 354
  348 C = - 1.0
      K = K + 1
      IF(K.LE.NSVP) GO TO 353
      K = K - 1
      IF(NOBOT)343,343,526
  353 GRAD = GI2
  354 IF(GRAD.NE.0.0) GO TO 373
      TATH = DABS(SITH/CSTH)
      IF(TATH.LT.0.001) GO TO 366
      DD = DABS(D(NR,K) - DEP(L-1))
      DRFT = DD/TATH
      DR = DRFT
      SITH2 = SITH
      CSTH2 = CSTH
      TANTH2 = TANTH
      ANGLE(L) = ANGLE(L-1)
      DEP(L) = D(NR,K)
      GO TO 401
  366 R(L) = R(L-1) + DELR
      DEP(L) = DEP(L-1)
      ANGLE(L) = 0.0
      SITH2 = SITH
      CSTH2 = CSTH
      TANTH2 = TANTH
      IF(DEP(L) - DEPMIN)514,414,414
  373 VS = VS + GRAD*(D(NR,K) - DEP(L-1))
      CSTH2 = SR*VS
      B = 1.0 - CSTH2**2
      Y = DSQRT(DABS(B))
      IF(Y.GE.0.001) GO TO 382
      CSTH2 = 1.0
      SITH2 = 0.0
      TANTH2 = 0.0
      GO TO 384
  382 IF(B.LT.0.0) GO TO 393
      SITH2 = C*Y
  384 DR = DABS((SITH - SITH2)/(SR*GRAD))
      ANGLE(L) = 57.29577951*DARSIN(SITH2)
      IF(DABS(ANGLE(L)).LE.85.0) GO TO 389
      NOANG = 1
      GO TO 527
  389 DEP(L) = D(NR,K)
      R(L) = R(L-1) + DR
```

```
      TANTH2 = SITH2/CSTH2
      GO TO 402
  393 KREST = 1
      DDEPTH = (1.0 - CSTH)/(SR*GRAD)
      VS = VS - GRAD*(D(NR,K) - DEP(L-1))
      IC = C
      K = K + IC
      ANGLE(L) = 0.0
      DEP(L) = D(NR,K) + DDEPTH
      DR = DABS((SITH/(SR*GRAD)))
  401 R(L) = R(L-1) + DR
  402 IF(DEP(L).GE.DEPMIN) GO TO 414
      L = L + 1
      IF(KREST.EQ.0) GO TO 515
  405 DEP(L) = D(NR,K)
      ANGLE(L) = - ANGLE(L-2)
      R(L) = R(L-1) + DR
      SITH2 = - SITH
      CSTH2 = CSTH
      TANTH2 = - TANTH
      IF(KREST.EQ.1) KREST = 0
      IF(DEP(L).GE.DEPMIN) GO TO 414
      GO TO 514
  414 DO 418 I = 2,NRBOT
      IF(R(L-1).GT.BRAN(I)) GO TO 418
      IBOT = I - 1
      GO TO 420
  418 CONTINUE
      GO TO 514
  420 RC = DABS(BRAN(IBOT) - R(L))
      RC2 = DABS(BRAN(IBOT+1) - R(L))
      IF(RC.LT.0.1.AND.DEP(L).EQ.BDEP(IBOT)) GO TO 425
      IF(RC2.GE.0.1) GO TO 427
      IF(DEP(L).NE.BDEP(IBOT+1)) GO TO 427
  425 NOM = 1
      GO TO 527
  427 ANGX = 0.01745329252*ANGLE(L-1)
      SITHX = DSIN(ANGX)
      CSTHX = DCOS(ANGX)
      TANTHX = SITHX/CSTHX
      IF(SITHX.LT.0.0) GO TO 435
      IF(DEP(L-1).GT.BDEP(IBOT)) GO TO 434
      IF(DEP(L-1).LT.BDEP(IBOT+1)) GO TO 511
  434 IF(SLOPE(IBOT))511,511,441
  435 IF(DEP(L).GT.BDEP(IBOT)) GO TO 441
      IF(DEP(L).GT.BDEP(IBOT+1)) GO TO 441
      IF(SLOPE(IBOT).EQ.0.0.AND.DEP(L).EQ.BDEP(IBOT)) GO TO 477
      IF(ANGLE(L).NE.0.0) GO TO 511
      KREST = 1
      GO TO 511
```

```
441 DM = BDEP(IBOT) - DEP(L-1) - SLOPE(IBOT)*(R(L-1) - BRAN(IBOT))
    IF(GRAD.NE.0.0) GO TO 446
    IF(SLOPE(IBOT).EQ.TANTHX) GO TO 511
    DELX = - DM/(TANTHX - SLOPE(IBOT))
    GO TO 458
446 GAM = 1.0/(SR*GRAD)
    ALPH = - GAM*SITHX
    BETA = - GAM*CSTHX
    AQ = SLOPE(IBOT)**2 + 1.0
    BQ = 2.0*(- SLOPE(IBOT)*(DM - BETA) - ALPH)
    CQ = ALPH**2 + (DM - BETA)**2 - GAM**2
    DISC = BQ**2 - 4.0*AQ*CQ
    IF(DISC.LT.0.0) GO TO 511
    IF(GRAD.GT.0.0) GO TO 457
    DELX = (- BQ + DSQRT(DISC))/(2.0*AQ)
    GO TO 458
457 DELX = (- BQ - DSQRT(DISC))/(2.0*AQ)
458 IF(DABS(DELX - DR).LE.0.1) GO TO 474
    IF(DELX.LT.0.0) GO TO 511
    XP = R(L-1) + DELX
    IF(XP.LT.BRAN(IBOT)) GO TO 511
    IF(XP.GT.BRAN(IBOT+1)) GO TO 511
    IF(XP.GT.R(L)) GO TO 511
    DELY = - DELX*SLOPE(IBOT) + DM
    R(L) = XP
    DEP(L) = DEP(L-1) + DELY
    VS = VS - GRAD*(D(NR,K) - DEP(L))
    CSTH2 = SR*VS
    IF(CSTH2.GT.1.0) GO TO 475
    IF(ANGLE(L-1).EQ.0.0) C = - C
    SITH2 = C*DSQRT(1.0 - CSTH2**2)
    TANTH2 = SITH2/CSTH2
    ANGLE(L) = 57.29577951*DARSIN(SITH2)
474 IF(DABS(ANGLE(L)).LE.85.0) GO TO 477
475 NOANG = 1
    GO TO 527
477 IF(R(L).GT.RMAX) GO TO 527
    IF(NOBOT.EQ.1) GO TO 527
    L = L + 1
    DEP(L) = DEP(L-1)
    R(L) = R(L-1)
    ANGLE(L) = 114.59156*DATAN(SLOPE(IBOT)) - ANGLE(L-1)
    IF(DABS(ANGLE(L)).LE.85.0) GO TO 486
    NOANG = 1
    GO TO 527
486 THONE = 0.01745329*ANGLE(L)
    SITH = DSIN(THONE)
    CSTH = DCOS(THONE)
    TANTH = SITH/CSTH
    IF(GRAD.EQ.0.0) GO TO 497
```

```
      IF(KREST.EQ.0) GO TO 495
      KREST = 0
      IF(SITH2.LE.0.0.AND.SITH.GT.0.0) K = K + 1
      GO TO 497
  495 IF(SITH.GT.0.0.AND.SITH2.GT.0.0) K = K + 1
      IF(SITH.LT.0.0.AND.SITH2.LT.0.0) K = K - 1
  497 L = L + 1
      SR = CSTH/VS
      IF(GRAD.NE.0.0) GO TO 330
      IF(SITH.GT.0.0.AND.SITH2.LT.0.0) K = K - 1
      DD = DEP(L-1) - D(NR,K)
      DRFT = DABS(DD/TANTH)
      DR = DRFT
      DEP(L) = D(NR,K)
      R(L) = R(L-1) + DR
      ANGLE(L) = ANGLE(L-1)
      SITH2 = SITH
      CSTH2 = CSTH
      TANTH2 = TANTH
      GO TO 514
  511 IF(R(L).LT.BRAN(IBOT+1)) GO TO 514
      IBOT = IBOT + 1
      GO TO 427
  514 L = L + 1
  515 IF(L.LT.4000) GO TO 518
      NOFIN = 1
      GO TO 526
  518 IF(R(L-1).GE.RMAX) GO TO 526
      IF(KREST.EQ.1) GO TO 405
      SITH = SITH2
      CSTH = CSTH2
      TANTH = TANTH2
      IF(R(L-1).LE.RSVP(NR+1)) GO TO 330
      NR = NR + 1
      IF(NR.LT.NRSVPS) GO TO 330
  526 L = L - 1
  527 IF(NAUT.EQ.0) GO TO 530
      DO 529 I = 1,L
  529 R(I) = 3.0*R(I)/6080.0
  530 IF(NOPR.GT.0) GO TO 549
      IB = 1
      NPAGE = 1
  533 WRITE(6,720) NPAGE,TITLE,SD,RA
      IF(NAUT.EQ.0) GO TO 537
      WRITE(6,721)
      GO TO 538
  537 WRITE(6,722)
  538 IL = IB + 43
      IF(IL.LE.L) GO TO 541
      IL = L
```

```
541 WRITE(6,723) (DEP(I),R(I),ANGLE(I), I = IB,IL)
    IB = IL + 1
    IF(IB - L)544,547,552
544 WRITE(6,710)
    NPAGE = NPAGE + 1
    GO TO 533
547 WRITE(6,723) DEP(L),R(L),ANGLE(L)
    GO TO 552
549 IF(NOMO.GT.1) GO TO 551
    WRITE(6,724) TITLE,SD
551 NOMO = NOMO + 1
552 IF(NOFIN.EQ.0) GO TO 554
    WRITE(6,725) RA
554 IF(NOM.EQ.0) GO TO 556
    WRITE(6,726) RA
556 IF(NOANG.EQ.0) GO TO 558
    WRITE(6,727) RA
558 WRITE(6,728) RA,L
    RMAXL = RMAX + 0.5*RANINC
    IF(R(L).LE.RMAXL) GO TO 566
561 DEP(L) = DEP(L-1) + (DEP(L) - DEP(L-1))*(RMAX - R(L-1))/(R(L) -
   1 R(L-1))
    R(L) = RMAX
    IF(R(L-1).LE.RMAX) GO TO 566
    L = L - 1
    GO TO 561
566 IF(NRAY.LT.2) GO TO 571
    IF(NOBOT.LT.1) GO TO 569
    L = L - 1
569 NRAY = 1
    GO TO 572
571 NRAY = 2
572 PRAN = R(L)/DELRAN + 0.05
    PDEP = - DEP(L)/DELDEP - 0.05
    IF(NRAY.LT.2) GO TO 601
575 XD = 200.0*R(1)/DELRAN
    YD = - 200.0*DEP(1)/DELDEP
    CALL DRAWQ(XD,YD,0)
    IB = 2
    IF(L.GT.990) GO TO 582
    IL = L
    GO TO 583
582 IL = 990
583 DO 587 I = IB,IL
    XD = 200.0*R(I)/DELRAN
    YD = - 200.0*DEP(I)/DELDEP
    CALL DRAWQ(XD,YD,1)
587 CONTINUE
    CALL GDMPQ
    IB = IL + 1
```

```
      IF(IB.GT.L) GO TO 595
      IL = IL + 990
      IF(IL.LE.L) GO TO 583
      IL = L
      GO TO 583
  595 IF(NRAY.LT.2) GO TO 270
  596 CALL XFSTQ(PRAN,PDEP,0.11,0.13,0.0,0.0,QD)
      CALL NMBRQ(RA,1,1,QD,GRAPH)
      CALL DISPQ(GRAPH,200.0)
      CALL REMVQ(GRAPH)
      GO TO (575,270),NRAY
  601 N = L/2
      DO 609 I = 1,N
      K = L - I + 1
      TR = R(I)
      TD = DEP(I)
      R(I) = R(K)
      DEP(I) = DEP(K)
      R(K) = TR
  609 DEP(K) = TD
      GO TO 596
  611 READ(5,704) DIST
      IF(DIST.LE.0.0) GO TO 615
      CALL ADPTQ(DIST,0.0,0,GRAPH)
      GO TO 616
  615 CALL ADPTQ(0.0,0.0,0,GRAPH)
  616 CALL DISPQ(GRAPH,200.0)
      CALL MOOVQ(200.0*DIST,0.0)
      CALL REMVQ(GRAPH)
      NOMO = 1
      IF(DIST.GT.0.0) GO TO 10
      WRITE(6,729)
      STOP
  701 FORMAT(40A1)
  702 FORMAT(I1,I3,2I1,F4.0,I2,3I1,3(I2,F8.2,F5.1),2F10.4)
  703 FORMAT(7I1)
  704 FORMAT(4F10.4,2I1)
  705 FORMAT(1H1,121X,'PAGE',I3,/1H0,88X,40A1,/89X'NUMBER OF POINTS IN',
     1 ' SVP',I2,' = ',I4)
  706 FORMAT(89X,'VELOCITIES COMPUTED')
  707 FORMAT(1H0,61X,'SVP ',I1,/1H0,33X,'DEPTH     TEMPERATURE',3X,
     1 'SALINITY   VELOCITY   VELOCITY GRADIENT'/35X,'(FT)',6X,
     2 '(DEG F)',6X,'(PPT)',5X,'(FT/SEC)',6X,'(FT/SEC/FT)' /)
  708 FORMAT(1H ,29X,F9.1,2F12.2,F13.3,/80X,F12.7)
  709 FORMAT(1H ,29X,F9.1,25X,F12.3,/80X,F12.7)
  710 FORMAT(1H0,58X,'(CONTINUED)')
  711 FORMAT(6F10.4)
  712 FORMAT(1H1,88X,40A1,/1H0,58X,'SVP RANGES'/1H0,55X,'SVP',8X,
     1 'RANGE'/68X,'(NM)')
  713 FORMAT(1H1,88X,40A1,/1H0,58X,'SVP RANGES'/1H0,55X,'SVP',8X,
```

```
    1 'RANGE'/67X,'(YDS)')
  714 FORMAT(1H0,55X,I2,F14.1)
  715 FORMAT(2F10.4)
  716 FORMAT(1H1,88X,40A1,/1H0,47X,'BOTTOM DEPTHS, RANGES AND SLOPES'
    1 /1H0,44X,'DEPTH',12X,'RANGE',12X,'SLOPE'/46X,'(FT)',7X,'(NM)',7X,
    2 '(YDS)' /)
  717 FORMAT(40X,F11.1,F10.1,F12.1,/73X,F11.4)
  718 FORMAT(F8.2,2I1,2F10.4)
  719 FORMAT(1H1,30X,'REQUESTED DEPTH OF',F7.1,' FT. IS GREATER THAN ',
    1 'LAST GIVEN SVP DEPTH OF',F7.1,' FT.')
  720 FORMAT(1H1,121X,'PAGE',I3,/1H0,88X,40A1,/89X,'SOURCE DEPTH  =',
    1 F8.1,' FT.'/89X,'INITIAL ANGLE =',F8.3,' DEG.'/1H0,56X,'SOUND RAY
    2 PATH'/1H0,49X,'DEPTH',7X,'RANGE',7X,'ANGLE')
  721 FORMAT(51X,'(FT)',8X,'(NM)',7X,'(DEG)' /)
  722 FORMAT(51X,'(FT)',7X,'(YDS)',7X,'(DEG)' /)
  723 FORMAT(45X,F10.1,F12.1,F12.2)
  724 FORMAT(1H1,88X,40A1,/89X,'SOURCE DEPTH =',F8.1,' FT.'/1H0,49X,
    1 'NUMBER OF POINTS IN RAY PATHS AND'/40X,'RAYS THAT TERMINATE ',
    2 'BEFORE REACHING DESIRED RANGE')
  725 FORMAT(1H0,40X,F5.1,' DEG. RAY TERMINATED. MORE THAN 4000 POINTS')
  726 FORMAT(1H0,40X,F5.1,' DEG. RAY TERMINATED. HIT BOTTOM BREAK.')
  727 FORMAT(1H0,40X,F5.1,' DEG. RAY TERMINATED. ANGLE GREATER THAN 85',
    1 ' DEG.')
  728 FORMAT(1H0,40X,'NUMBER OF POINTS IN',F5.1,' DEG. RAY =',I5)
  729 FORMAT(1H1,50X,'RUN COMPLETED')
      END
```

Appendix C

The input for the eigenray and ray-tracing programs is described here. First, the input to the eigenray routine contained in Appendix A is given.

Eigenray Program Operation - Sequence of Data Cards

Columns			Format
1. Title card			
1-40	title		10A4
2. Sound velocity profile control card			9I1
1	NDRT = 0 / 1	depth given in meters / feet	
2	NVFT = 0 / 1	sound velocity in m/sec / ft/sec	
3	NTF = 0 / 1	temperature given in degrees C / F	
4	NWV = 0 / 1	wind velocity given in m/sec / ft/sec	
5	NRFT = 0 / 1	range given in meters / feet	
6	IVEL = 0 / 1	sound velocity calculated / given	
7	ITEMP = 0 / 1	temperature not given / given	
8	IWV = 0 / 1	wind velocity not given / given	
9	IRHM = 0 / 1	relative humidity not given / given	
3. Sound velocity profile data cards - must be given as depths from highest level to ground surface			4F10.4,1x,I1
1-10	DEP	- depth	
11-20	TEMP	- temperature	
21-30	VEL	- sound velocity	
31-40	WV	- wind velocity	
42	NOMO	=1 to indicate last SVP data card	

4. Ground loss card			I1,4X,2F5.1
1	NBL	- number of runs with different ground losses	
6-10	POR	- porocity of ground	
11-15	BL	- ground loss coefficient	
5 Time integration window card			F10.3
1-10	TWIN	- time window	
6. Ray path parameter card			7F10.3
1-10	SD	- source depth	
11-20	TD	- target depth	
21-30	RANGE	- range from source to receiver	
31-40	ANGMAX	- maximum initial ray angle in degrees	
41-50	ANGMIN	- minimum initial ray angle	
51-60	FREQ	- frequency	
61-70	RHM	- relative humidity	

All input must be consistant with the units specified in the control card (2).

The input for the ray-tracing routine is given here. It is noted that the ray-tracing routine graphics are system dependent. The data to be plotted is output using the following packages: XFSTQ,LABLQ, AXISQ, DISPQ, REMVQ, EDINCH, ADPTQ, DRAWQ, GDMPQ, NMBRQ, MOOVQ. These are packages available in the PSU computer center's accessible library. The output is then plotted onto a Tektronix 4662 plotter using the package CONTK.

Ray-Tracing Program Operation - Sequence of Data Cards

Columns		Format
1. Title card		
1-40	title	40A1

AD-A108 626 PENNSYLVANIA STATE UNIV UNIVERSITY PARK NOISE CONTROL LAB F/G 20/1
RAY TRACING TECHNIQUES - DERIVATION AND APPLICATION TO ATMOSPHE--ETC(U)
JAN 80 S D ROTH DAAK07-80-C-0001

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2 2

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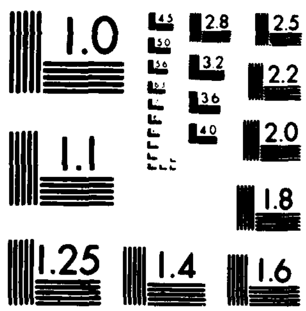

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

2. Sound velocity profile control card

1	NRSVP	- number of SVP's - maximum of 5	I1
2-4	NRBOT	- number of ground height coordinates - maximum of 150 minimum of 2	I3
5	METER = 1	all heights given in meters	I1
6	NAUT = blank	all ranges given in meters	I1
7-10	DELR	- distance ray travels when ray angle is zero and there is no refraction	F4.0
11-12	NOUT = 7)		I2
13	NEWSVP =	blank - uses SVP's from previous run 1 - new SVP's	I1
14	NOPR =	blank prints / 1 does not print ray paths	I1
15	NOAD =	blank writes "SOUND RAY PATHS" / 1 writes "SOUND RAY PATHS AND INTENSITY CONTOURS" and Alpha headings on plots	I1
16-17	NRAN	- number of divisions on range scale	I2
18-25	RANINC	- increment on range scale - meters	F8.2
26-30	RANL	- length in inches of range scale	F5.1
31-32	NDEP	- number of divisions on height scale	I2
33-40	DEPINC	- increment on height scale - meters	F8.2
41-45	DEPL	- length in inches of height scale	F5.1
46-47	NSV	- number of divisions on SVP scale	I2
48-55	SVINC	- increment on SVP scale - m/sec	F8.2
56-60	SVL	- length in inches of SVP scale	F5.2
61-70	SVMIN	- minimum value on SVP scale (generally 335 m/sec)	F10.4
71-80	RANGE	- range to reference line in meters (if blank-no reference line printed)	F10.4

3. Alpha heading card (omitted if NOAD is blank)

1-20	ALPD	- graph heading	20A1

One set of cards 4 and 5 must be given for each sound velocity profile

4. SVP control card 7I1

1	NODEP = blank	
2	NODFT = 1	
3	NOTEMP = blank	temperature given
	1	sound velocity in still air given
4	NOTF = 1	
5	NOVEL = blank	
6	NOVFT = 1	
7	NOSAL = blank	

5. Sound velocity profile cards - maximum of 250 for each SVP 4F10.4,2I1

1-10	D - height	meters	
11-20	TF - temperature	Celsius	
21-30	WV - wind velocity	m/sec	
31-40	SAL - blank		
41	IGSVP = blank	interpolated	SVP value
	1	given	
42	NOMO - blank	indicates	not last SVP card
	1		last SVP card

6. Location of SVP's card (first always 0.0) 5F10.4
(omitted if only one SVP)

1-10	RSVP(1) = 0.0	
11-20	RSVP(N) - locates SVP	meters

etcetera

7. Ground height coordinate cards (must be NRBOT cards, value given on card 2) (must be in order of increasing range) 2F10.4

1-10	BDEP - height	meters
11-20	BRAN - range	meters

8. Ray parameter cards F8.2,2I1,2F10.4

1-8	SD - source height	meters
9	NOSUR = 1	
10	NOBOT = blank	
11-20	RA - initial angle in degrees	
21-30	RMAX - maximum range in meters	

9. Blank card

END

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